Original article

The new method on gas-water two phase steady-state productivity of fractured horizontal well in tight gas reservoir

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Abstract: Based on a large number of tight gas exploration and development literature researches, it is found that the horizontal drilling technology and fracturing operation are usually used in the tight gas reservoir for its low permeability. Three tasks have been done in this paper. First, we described the characteristics and the flow mechanism of the tight gas reservoirs which are different from the conventional ones and gave a new definition of two phase pseudo pressure. Second, formation stress sensitivity, starting pressure gradient and the interaction of fractures are considered. Equivalent flow resistance was used to establish a model of tight gas steady flow, and a new productivity formula of fractured horizontal well in tight gas reservoir derived by the new flow model. Third, the productivity influence research has been done, which gives the influence degree of different parameters. It is signified that the productivity will increase with the addition of the permeability modulus and decrease with the increase of the water-gas volume ratio. The influence of starting pressure gradient was not very clear and the fractures parameters showed the opposite situation. Therefore optimization design of tight gas reservoir development mode can be improved by the productivity formula and research.

Keywords: Tight gas, fracture, horizontal, gas-water two phase, steady-state productivity.

1. Introduction

Recently, the exploration of the tight gas reservoirs has drawn an extremely attention, which are regarded as the potential unconventional resources all over the world (Lin et al., 2017; Xiong et al., 2017). Compared with the conventional ones, the flow mechanism is more complex, which means greater development difficulties and higher technical requirements for its specific accumulation model and reservoir characteristics (Xie et al., 2014; Damjanac and Cundall, 2015). Currently, there has had extensive research on the cause and geological characteristics and exploration technics of the tight gas reservoir (Raghavan and Chen, 2013; Siavoshi and Bahrami, 2013; Wang, 2014). However, the research on steady-state productivity model based on its specific flow mechanism is still very little, and the study method of gas-water two phase productivity considering the water influence is seldom reported (Brown et al., 2011; Ozkan et al., 2011; Zhao et al., 2014). On the basis of traditional fractured horizontal well model, considering the stress sensitivity (Avseth et al., 2009), starting pressure gradient (Kutilek, 1972), inducing the new definition of gas-water two phase pseudo pressure, using equivalent flow resistance principle, this paper deduced analytical solution for this productivity model and analyzed the influence of permeability modulus, starting pressure gradient and the fracture parameters on production (Al-Khidir et al., 2013; Ibrahim et al., 2012; Escobar et al., 2014). It can not only provide new theoretical basis for the exploration of tight gas reservoirs, but also give a new idea for the solution of the gas-water two phase flow productivity in unconventional gas reservoir (Gao et al., 2013).

It is generally thought that the stress sensitivity exits in the low permeability reservoir formation, but the effect of slippage effect, starting pressure gradient and high speed non-Darcy flow in this reservoir which is mainly associated with the water saturation, permeability and flow velocity is still in dispute (Tan et al., 2015a, 2015b). Because of the high

water saturation, the low porosity and the low flow velocity in low permeability water cut tight gas reservoir, the influence of starting pressure gradient should be taken into account and the effect of slippage effect and high speed non-Darcy flow could be neglected (Cai, 2014; Cai et al., 2014).

2. Definition and solution of gas-water two phase pseudo pressure

Based on the motion equation, the gas and water mass flow rates are as follows (Sánchez-Palencia, 1980):

$$m_g = \rho_g q_g = \frac{K_i exp\left(-\alpha(p_i - p)\right)K_{rg}\rho_g}{\mu_g}\left(\frac{dp}{dr} - \lambda_g\right) \quad (1)$$

$$m_w = \rho_w q_w = \frac{K_i exp\left(-\alpha(p_i - p)\right)K_{rw}\rho_w}{\mu_w}\left(\frac{dp}{dr} - \lambda_w\right) \quad (2)$$

The volume ratio of water and gas is defined as: $R_{wg} = q_w/q_g$. Based on the mass conservation principle, the gas-water two phase mass flow rate is as follows (Bear, 1972):

$$m_t = \left(R_{wg}\rho_w + \rho_{gsc}\right)q_{gsc} \quad (3)$$

For the convenience of integration, the new gas-water two phase pseudo pressure considering the stress sensitivity is defined as

$$\varphi(p) = \int_0^p \left[\frac{\rho_g K_{rg}exp\left(-\alpha(p_i - p)\right)}{\mu_g}\right. $$
$$\left. + \frac{\rho_w K_{rw}exp\left(-\alpha(p_i - p)\right)}{\mu_w}\right]dp \quad (4)$$

The two phase pseudo starting pressure gradient is defined as (Boukadi et al., 1998)

$$\lambda_{\varphi m} = \frac{\rho_g K_{rg}exp\left(-\alpha(p_i - p)\right)}{\mu_g}\lambda_g $$
$$+ \frac{\rho_w K_{rw}exp\left(-\alpha(p_i - p)\right)}{\mu_w}\lambda_w \quad (5)$$

Substituting the pressures with p_e and p_{wf} respectively, the difference formula of new pseudo pressure is as follow

$$\varphi(p_e) - \varphi(p_{wf}) = \int_{r_{wf}}^{R_e} \lambda_{\varphi m}dr + \int_{r_{wf}}^{R_e} \frac{\left(R_{wg}\rho_w + \rho_{gsc}\right)q_{gsc}}{AK_i}dr \quad (6)$$

The $\int_{r_{wf}}^{R_e} \lambda_{\varphi m}dr$ can be obtained by the trapezoid formula and kept as I for short to get the simplified formula. The viscosity and density of water can be viewed as constants for that K_{rw}, K_{rg}, μ_g, ρ_g are given as a function of pressure in Eq. (4). Because the value of λ_g and λ_w is very small, using relative permeability curves, the relationship between K_{rw} and K_{rg} can be obtained as follow (Craft et al., 1991):

$$\frac{K_{rw}}{K_{rg}} = \frac{R_{wg}\mu_w}{\mu_g} \quad (7)$$

Since the volume ratio of water and gas is supposed as a constant, the relationship between the ratio of water and gas two phase relative permeability K_{rw}/K_{rg} and pressure p can be determined. The relationship between the two phase relative permeability and the pressure can be deduced on the basis of the relationship between the pressure and the water saturation which can be obtained from the phase percolation curves. The gas-water two phase pseudo pressure in specific pressure can be obtained by taking the relationship into Eq. (4) and doing the numerical integration.

3. Deliverability equation of gas water two phase flowing in tight gas reservoir

One of domestic scholars used the capsule model to simulate the drainage area around the horizontal well. The flow region area around the horizontal well can obviously increase for the hydraulic fracture. In conclusion, the flow region of fractured horizontal well in tight gas reservoir can be considered to be a big capsule which constitutes two hemispheroids and a cylinder (Lips and Meyer, 2011).
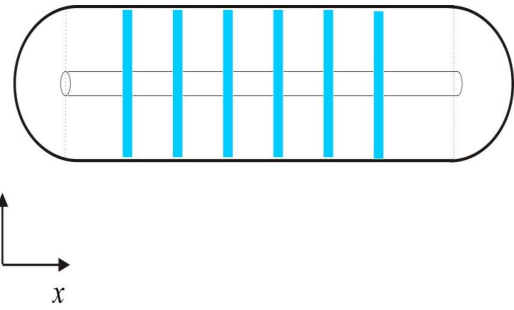

Fig. 1. Capsule flow region of fractured horizontal well in tight gas reservoir.

As shown in Fig. 2, the steady flow of fractured horizontal well in tight gas reservoir has been divided into three parts:

(1) Gas from the flow region of horizontal well flows into fractures from matrix, and then flows into horizontal well;

(2) Gas from the flow region of horizontal well flows into horizontal well by radial flow;

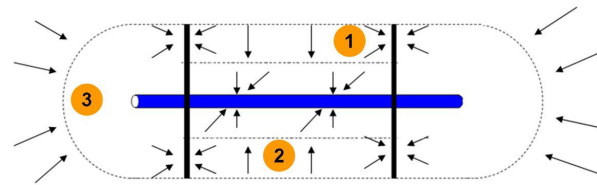(3) Gas from both ends of the flow region of horizontal well flows into horizontal well by spherical centripetal flow.

Fig. 2. Fractured horizontal well flow in tight gas reservoir.

Assuming the vertical fractures go across the horizontal well, principle of area equivalence was used and a vertical
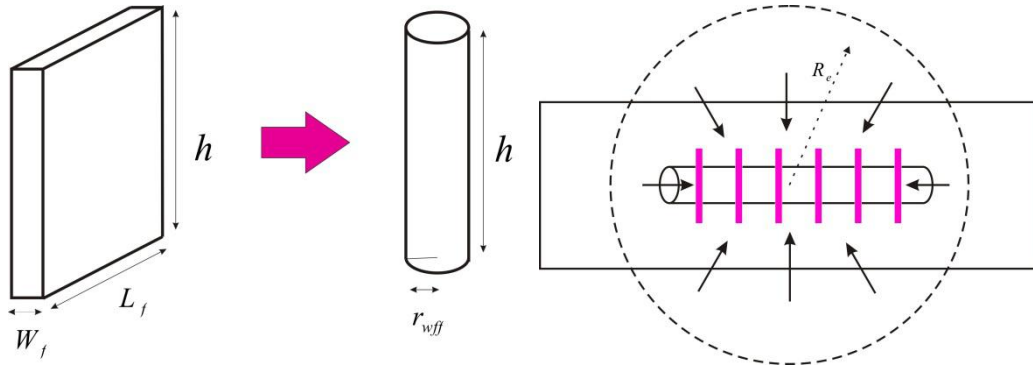
Fig. 3. Principle of area equivalence and equivalent pseudo circle flow area.

fracture flow was considered as a vertical well flow. Fractures interference can also be expressed by mirror reflection model using superposition principle.

Based on the definition of velocity potential, transforming the quasi-rectangular part into quasi-circular one in the capsule model, the potential of any place in this reservoir can be expressed as (Tan et al., 2014)

$$\varphi = \frac{Q_{mix}}{2\pi HK_i\rho_m}\ln r + \int_0^r \frac{\lambda_{\varphi m}}{\rho_m}dr + C \tag{8}$$

Supposing the fractures as a well array, the difference of the pseudo pressure can be obtained by using the superposition principle as follows:

$$\varphi(p_e) - \varphi(p_{wf}) = (G_2 - G_1) + \frac{(R_{wg}\rho_w + \rho_{gsc})q_{gsc}}{4\pi hK_i\rho_m}$$
$$[\ln(ch\frac{\pi R_e}{a}+1) - \ln(1-\cos\frac{\pi r_{wff}}{a})] \tag{9}$$

$$G_2 - G_1 = \frac{N}{4\rho_m}\left(\sqrt{[(\frac{1}{2}L)^2 + R_e^2]} + R_e\right)$$
$$\left[\left(\frac{\rho_g K_{rg}exp(-\alpha(p_i - p_e))(1+\frac{b_g}{p})}{\mu_g}\lambda_g\right.\right.$$
$$+ \frac{\rho_w K_{rw}exp(-\alpha(p_i - p_e))}{\mu_w}\lambda_w\right)$$
$$+ \left(\frac{\rho_g K_{rg}exp(-\alpha(p_i - p_{wf}))(1+\frac{b_g}{p})}{\mu_g}\lambda_g\right.$$
$$+ \left.\left.\frac{\rho_w K_{rw}exp(-\alpha(p_i - p_{wf}))}{\mu_w}\lambda_w\right)\right] \tag{10}$$

Therefore, the fracture productivity in the form of potentials can be simplified as:

$$q_f = \frac{4\pi K_i H\rho_m\left(\varphi(p_e) - \varphi(p_{wf}) - (G_2 - G_1)\right)}{\left(\frac{\pi R_e}{2a} + \ln\frac{a}{\pi r_{wff}}\right)\cdot(R_{wg}\rho_w + \rho_{gsc})} \tag{11}$$

Assuming that the distance between the first fracture and the boundary is the same as the distance between the last frac-

ture and the boundary, 2a is cracking space, N is the number of fractures. Total fractures productivity can be obtained as

$$Q_N = \frac{4\pi NK_i H\rho_m\left(\varphi(p_e) - \varphi(p_{wf}) - (G_2 - G_1)\right)}{\left(\frac{2\pi R_e}{(N+1)L} + \ln\frac{(N+1)L}{4\pi r_{wff}}\right)\cdot(R_{wg}\rho_w + \rho_{gsc})} \tag{12}$$

Pressure drop funnel exists in gas flow, especially in the horizontal well. The smaller the flow region is, the bigger the percolating resistance becomes. There are some viewpoints which believe that different areas in the horizontal well have different flow states. However, the research emphasis is about the gas-water two phase flow. Approximate treatment was made in this paper and only the radial flow during the gas flowing into the wellbore was considered.
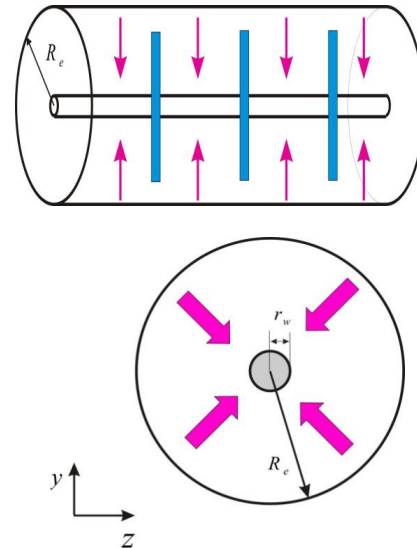

Fig. 4. Horizontal well radial flow.

The productivity expression can be obtained by the radial flow formula as follows:

$$q_2 = q_{gsc} = \frac{2\pi LK_i[(\varphi(p_e) - \varphi(p_{wf})) - 1]}{(R_{wg}\rho_w + \rho_{gsc})\ln\frac{R_e}{r_w}} \tag{13}$$

The last flow part is spherical centripetal flow which flowing from both ends of the horizontal well and ending as shown in Fig. 5
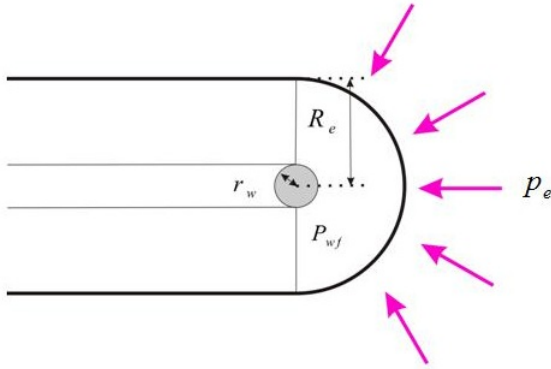

Fig. 5. Spherical centripetal flow.

By using the spherical centripetal flow equation with the solution of pseudo pressure, the spherical centripetal flow productivity equation is deduced as follows:

$$q_3 = q_{gsc} = \frac{2\pi K_i[(\varphi(p_e) - \varphi(p_{wf})) - 1]}{(r_w^{-1} - r_e^{-1})(R_{wg}\rho_w + \rho_{gsc})} \tag{14}$$

The productivity of horizontal well is usually divided into different parts. The solution is presented by equivalence percolation resistance law. All seepage stages' percolating resistance can be divided into three types: (1) Matrix and fracture percolating resistance Ω_1. (2) Radial flow percolating resistance Ω_2. (3) Spherical centripetal flow percolating resistance Ω_3. Total percolating resistance can be written as follows:

$$
\begin{aligned}
\frac{1}{\Omega_{sum}} &= \frac{1}{\Omega_1} + \frac{1}{\Omega_2} + \frac{2}{\Omega_3} \\
&= \frac{4\pi N K_i H \rho_m(\varphi(p_e) - \varphi(p_{wf}) - (G_2 - G_1)\varphi)}{\left(\frac{2\pi R_e}{(N+1)L} + \ln\frac{(N+1)L}{4\pi r_{wff}}\right)(R_{wg}\rho_w + \rho_{gsc})(\varphi(p_e) - \varphi(p_{wf}))} \\
&+ \frac{2\pi L K_i((\varphi(p_e) - \varphi(p_{wf})) - 1)}{(R_{wg}\rho_w + \rho_{gsc})\ln\frac{R_e}{r_w}(\varphi(p_e) - \varphi(p_{wf}))} \\
&+ \frac{4\pi K_i((\varphi(p_e) - \varphi(p_{wf})) - 1)}{(r_w^{-1} - r_e^{-1})(R_{wg}\rho_w + \rho_{gsc})(\varphi(p_e) - \varphi(p_{wf}))}
\end{aligned}
\tag{15}
$$

Employing $Q = \Delta\varphi/\Omega$, the production equation can be obtained as follows:

$$
\begin{aligned}
Q_{sum} &= \frac{\phi(p_e) - \phi(p_{wf})}{\Omega_{sum}} = \frac{P}{F}\left(\frac{2NH\rho_m(P - (G_2 - G_1))}{\left(\frac{2\pi R_e}{(N+1)L} + \ln\frac{(N+1)L}{4\pi r_{wff}}\right)P}\right. \\
&\left. + \frac{L(P-1)}{P\ln\frac{R_e}{r_w}} + \frac{2(P-1)}{P(r_w^{-1} - R_e^{-1})}\right) = \frac{1}{F}\left(\frac{2(P-1)}{(r_w^{-1} - R_e^{-1})} + \right.\\
&\left. \frac{2NH\rho_m(P - (G_2 - G_1))}{\left(\frac{2\pi R_e}{(N+1)L} + \ln\frac{(N+1)L}{4\pi r_{wff}}\right)} + \frac{L(P-1)}{\ln\frac{R_e}{r_w}} + \right)
\end{aligned}
\tag{16}
$$

where, $F = (R_{wg}\rho_w + \rho_{gsc})/2\pi K_i$, $P = \varphi(p_e) - \varphi(p_{wf})$.

4. Examples and productivity influence research

According to the data of an actual gas horizontal well in domestic tight gas reservoir, we programmed to calculate the output by changing one parameter and fixing the others, then the productivity influence analysis using IPR curves was done. Table 1 shows the data of the tight gas reservoir and fracture parameters.

Table 1. Parameters of tight gas reservoir and fractures.

Parameter	Value
Seepage region (m^3)	9.8×10^5
Reservoir length (m)	1,468
Reservoir width (m)	668
Initial reservoir pressure (MPa)	28.4
Supply boundary pressure (MPa)	26.619
Bottomhole pressure (MPa)	21.572
Horizontal well radius (m)	0.1
Horizontal well length (m)	561
Formation temperature (K)	353
Gas viscosity (mPa·s)	0.04
Deviation factor	0.9555
Fracture length (m)	100
Fracture number	6
Fracture interval (m)	110

Fig. 6 shows that the stress sensitivity has a big effect on the productivity. The bigger the permeability modulus is, the lower the productivity becomes. The stress sensitivity effect will become more obvious with the increase of the differential pressure of production. Fig. 7 shows that the gas-water two phase flow state have a large effect on the productivity which expresses that the bigger the water-gas ratio is, the lower the productivity is. This is because that large space in the pore is occupied by the water which diminishes the seepage space of the gas flow. Fig. 8 shows that with the increase of the starting pressure gradients of gas and water, the decrease of the production is not very clear. The curve tendency indicates that the starting pressure gradient has little effect on the productivity which could be simplified in the forecasting model.

Figs. 9-11 show that the parameters of horizontal length and fractures, which are considered to be the main influence factors of the deliverability, have a big effect on the productivity. The longer the horizontal well is, the bigger the flow area becomes, which indicates higher productivity. When the number of fractures increases and permeability of fractures increases, the productivity will also increase.
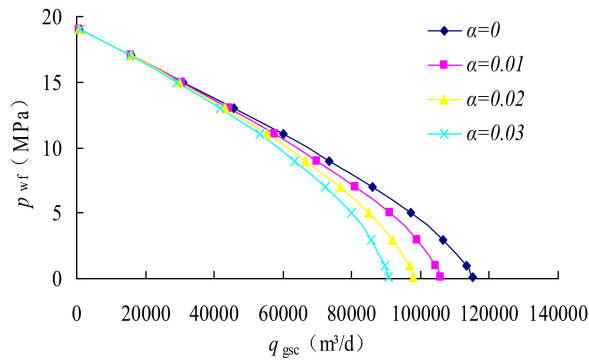
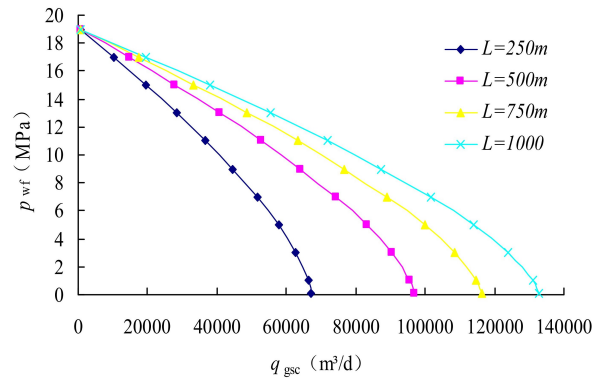**Fig. 6.** Permeability and porosity relationship for the tested core samples.

Fig. 7. Permeability and porosity relationship for the tested core samples.

Fig. 8. Permeability and porosity relationship for the tested core samples.

Fig. 9. Permeability and porosity relationship for the tested core samples.

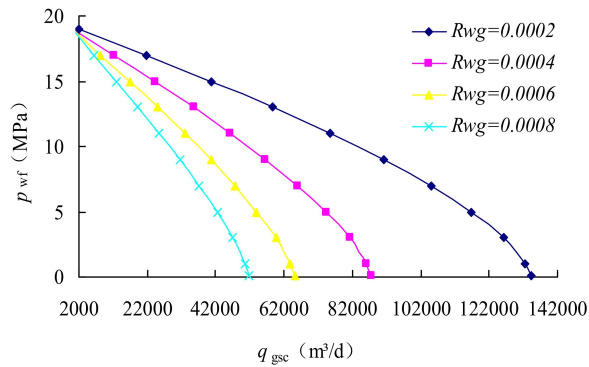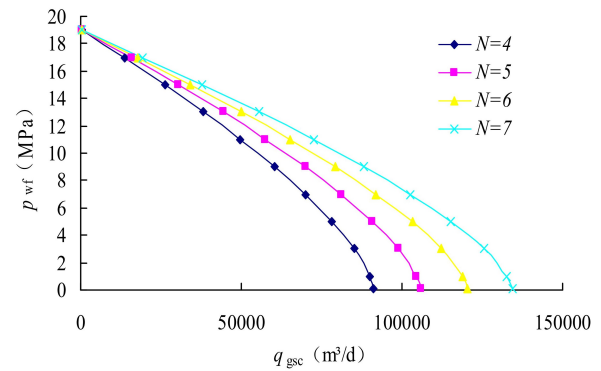**Fig. 10.** Permeability and porosity relationship for the tested core samples.

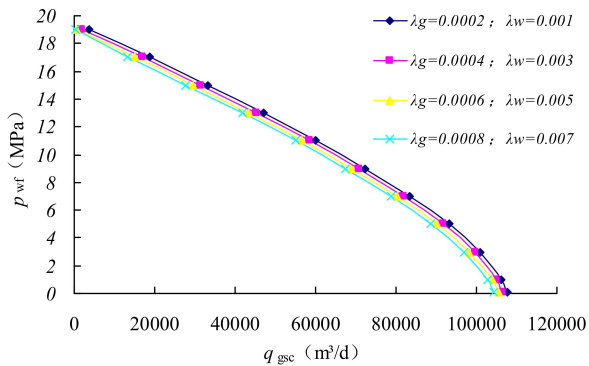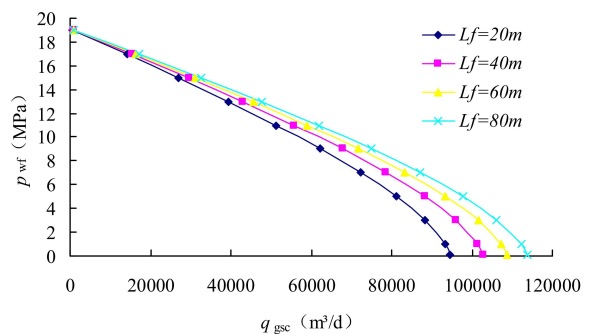**Fig. 11.** Permeability and porosity relationship for the tested core samples.

5. Conclusion

In this article, a steady flow model for fractured horizontal wells in tight gas reservoir was presented. The influences of hydraulic fractures, reservoir properties and horizontal well parameters were discussed.

Stress sensitivity and starting pressure phenomenon usually exist in the tight gas reservoir for its specific flow mechanism. Based on the theory of solution in conventional reservoirs, defining the two phase pseudo pressure expression with stress sensitivity and starting pressure gradient, using the potential function principle, considering the interference between fractures, this article deduced an analytical model for gas-water two phase flow productivity in fractured horizontal well of the tight gas reservoir.

Numerical integration was applied in this article to solve the pseudo pressure. The relationship of permeability and pressure provides new ideas for the solution in gas-water two phase flow productivity.

The examples and productivity influence analysis indicated that the effect of the gas-water two phase starting pressure gradient is not very clear which can be neglected in the deliverability forecasting model. However, permeability module, water-gas volume ratio and the corresponding changes of fracture parameters have a big effect on the productivity which need to be paid more attention to the control of formation water production and the fracture parameters optimization.

Nomenclature

p_i = Initial reservoir pressure (MPa)
p_{wf} = Bottom hole pressure (MPa)
p_e = Supply boundary pressure (MPa)
K_i = Initial formation permeability (m^2)
α = Permeability module (Pa^{-1})
ν = Flow rate (m/s)
μ = Viscosity (mPa·s)
p = Average formation pressure (MPa)
m_t = Two phase mass flow rate (kg/s)
m_g = Gas mass flow rate (kg/s)
m_w = Water mass flow rate (kg/s)
q_g = Gas volume flow rate (m^3/d)
q_w = Water volume flow rate (m^3/d)
q_{gsc} = Surface gas production rate (m^3/d)
q_{wsc} = Surface water production rate (m^3/d)
ρ_{gsc} = Surface gas density (kg/m^3)
ρ_{wsc} = Surface water density (kg/m^3)
ρ_m = Mixed density (kg/m^3)
r_{wg} = Water-gas volume ratio
A = Flow region area (m^2)
R_e = Gas drainage radius (m)
r_w = Radius of horizontal well (m)
r_{wff} = Equivalent well radius (m)
K_{rw} = Relative water permeability, dimensionless
K_{rg} = Relative gas permeability, dimensionless
λ_g = Gas starting press gradient (Pa/m)
λ_w = Gas starting press gradient (Pa/m)
$\lambda_{\varphi m}$ = Pseudo two phase starting pressure gradient (Pa/m)
d = Hydraulic fracture half interval (m)
L_f = Hydraulic fracture length (m)
N = Fractures number, dimensionless
L = Horizontal well length (m)
T = Formation temperture (K)
Z = Z-factor

Acknowledgments

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