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Original article

A phenomenological description of the transient single-phase pore velocity period using the resistance force-velocity relationship

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Abstract:
A new approach to determine the transient period towards steady state pore flow velocity for fluids propagating through porous media under constant pressure condition is presented. The transient expression relates to the mean pore velocity rather than the fluid pressure conventional considered when characterizing transient behavior in porous media. It is based on the general, resistance force-velocity relationship, and is therefore analogous to the approach used when calculating transient periods for objects falling through resisting liquid fluids and for the increase in electric currents towards respective steady state values. The transient is caused by inertia forces and characterized by a relaxation time comprising fluid density and viscosity together with porous medium properties as porosity and absolute permeability. Results show that the transient period increases with decreasing medium porosity and fluid viscosity and with increasing fluid density and absolute permeability of the medium. The transient period is negligibly small for typical fluid/medium property values characterizing typical subterrain sandstone reservoirs. Significant transient periods, occasionally observed during laboratory fluid injection tests, are therefore caused by other time-dependent processes not captured by the transient expression presented herein, e.g., fines migration or electrokinetic phenomena.

1. Introduction

Darcy's law (Darcy, 1856; Bear, 1972; Dullien, 1975) describes the relationship between applied pressure and fluid flux for single-phase flow through porous media when steady state occurs. It is extensively used in many applied sciences, e.g., soil and material sciences and chemical- and petroleum engineering. The law was formulated based on empirical observations related to vertical water flow through sandpacks. Darcy's law solved for the external pressure ($-\Delta P$), required to force fluids through homogenous porous media at low rates under steady laminar flow conditions reads,

$$-\Delta P=\frac{\mu L}{K}v=\frac{\phi\mu L}{K}u_P \tag{1}$$

where K is the absolute permeability of the medium (referred herein to as permeability only), ϕ is the porosity, L is the length, μ is the fluid viscosity, v is the fluid flux (Darcy velocity) and u_P is the mean linear velocity or mean pore velocity. The relationship between the Darcy- and the mean pore velocity is $v=\phi u_P$. Darcy's law can be derived from the Navier-Stokes equation via Stokes flow for slow laminar flow as shown in several works (Foster et al., 1967; Slattery, 1969; Neuman, 1977; Whitaker, 1986), by assuming incompressible medium and fluids and by neglecting the non-linear velocity term as well as the inertia term. Since Darcy's law only is valid under steady state flow condition, a transient period will therefore occur where the instantaneous pore fluid velocity will increase from zero to its steady value given by Darcy's law. Initial periods characterized by instabilities are occasionally experienced upon when injecting water through porous media (Brace et al., 1968; Yang et al., 2019). A new phenomenological approach to derive an expression for the

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transient pore velocity build-up period is the subject of the current paper. It is based on the same approach applied when describing e.g., the transient build-up of an electrical current in circuits (Smith, 1984) or the increase in velocity of falling objects towards a steady terminal velocity (Alonso and Finn, 1983). In these cases, the steady state condition is described by Ohm's law and by applying Newton's second law to a falling object including liquid fluid resistance, respectively. The common physical characteristics of the processes are that a step value (Heaviside function) of the external potential is initiated at time zero. The resulting force, constant for all subsequent times, is then balanced by a resistance force assumed proportional to the velocity of the electrical charge carriers or the object, plus an additional inertia term. The latter therefore decreases as a function of time due to the increase in velocity and vanishes at steady state condition. The principle which connects the description of all these cases is referred to as the resistance force-velocity relationship described in the next section and exemplified considering the falling object case.

The aim of the paper is therefore to determine a relaxation time expression, which characterizes the transient period towards a steady state pore velocity value given by Darcy's law. It is important to emphasize that the transient period discussed is related to the pore fluid velocity developed under constant external pressures conditions on the inlet and outlet boundaries. The approach differs from the conventional way transient behavior in porous media have been reported in the literature, which focuses on the propagation of pressure pulses through the fluid-saturated medium (Foster et al., 1967; Odeh and McMillen, 1972; Taherkhani and Pourafshary, 2012; Zimmerman, 2018). The transient pressure pulse period is described by a diffusion equation in the limiting case of an infinite pressure pulse speed (Zimmerman, 2018) or by a modified diffusion equation, referred to as the telegrapher's equation (Foster et al., 1967; Taherkhani and Pourafshary, 2012) originally developed by Heaviside for transmission of electrical signals, for finite pressure pulse propagation. Differences between the pressure pulse approach and the one presented herein will appear if some of the quantities which relate pressure to pore fluid velocity happen to be velocity or shear-rate dependent. Hence, an advantage of the current approach is therefore that it potentially also can capture such effects, e.g., if the fluid at hand is viscoelastic. Background for the analogue description outlined above is presented in the next section, before presenting the expression, which characterizes the transient pore velocity build-up period for single-phase fluid flow in porous media. It should finally be mentioned that the approach presented is phenomenological since no detailed description of the underlying mechanisms determining the permeability of the medium is considered. A more detailed discussion regarding these issues will, however, be provided in the Discussion section.

1.1 The resistance force-velocity relationship

The applicability of the falling object description to single-phase fluid flow in porous media is based on the observation that only relative velocities between the object and the fluid is important when calculating resistance forces. The famous Stokes' law for the viscous resistance force on fluids flowing slowly around spherical particles ($F_{Stokes} = 6\pi\mu R_S v_R$, R_S is the radius of the sphere and v_R is the relative velocity) can e.g., be derived from the reference frame where a fluid is passing a stagnant sphere (Stokes, 1851; Dey et al., 2019) or where a spherical particle is moving through a stagnant fluid (Landau and Lifshitz, 1987; Batchelor, 2000). Resisting frictional forces can therefore always be determined by an expression on the form,

$$\text{Resistance force } = \gamma \cdot \text{ relative velocity} \tag{2}$$

where γ is a friction coefficient, regardless of whether the object is moving, and the fluid is stagnant, or vice versa. The validity of Eq. (2) is of course based on the assumption of a direct proportionality between resistance force and relative velocity, i.e., for low relative motions. This important and powerful expression will be referred to as the resistance force-velocity relationship. It allows for analysis of transient periods for both cases mentioned above using the same approach. The only difference is that a force balance on the fluid must be considered for porous media flow whereas a force balance on the object is considered for the falling object case. The calculation of the resistance force, however, remains the same in both reference frames since it is proportional to the relative velocity. The resistance force-velocity relationship therefore connects the description of transient fluid flow in porous media to any other phenomenon where frictional forces are determined by relative velocities. It is general and therefore powerful as additional resistance terms easily can be included to obtain a total flow resistance comprising several different contributions as discussed in Standnes (2022).

Consider first the description of a spherical object with net mass M (mass of the object corrected for the buoyancy of the fluid) falling towards earth accounting for fluid resistance. Choosing positive direction downwards and assuming the acceleration due to gravity to be constant, Newton's 2nd law gives by considering the force balance on the object (Alonso and Finn, 1983),

$$Mg - \gamma u(t) = M\frac{du(t)}{dt} \tag{3}$$

The body force on the object is represented by the Mg term, where g is the acceleration of gravity and the resistance force caused by the surrounding fluid given by the term $\gamma u(t)$. The development towards a steady state terminal velocity is then given by the well-known expression,

$$u(t) = \frac{Mg}{\gamma}\left(1 - \mathrm{e}^{-\frac{\gamma}{M}t}\right) \tag{4}$$

The steady state terminal velocity, Mg/γ, occurs when the contribution from the acceleration term, $du(t)/dt$, has vanished. That is, for times so large that the exponential term inside the bracket in Eq. (4) is negligible, typically for $(\gamma/M)t > 4$, at which 98.2% of the steady state velocity value has been obtained. The situation is illustrated in Fig. 1, where the transient velocity of the object towards steady state termin-

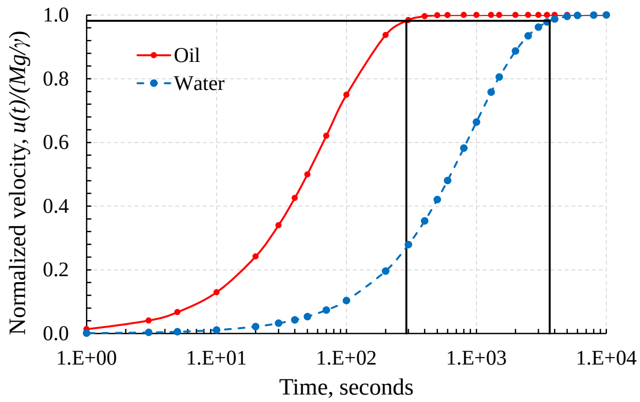

Fig. 1. Transient period towards a steady state terminal velocity for a 5 cm spherical particle falling through water and oil, respectively. The vertical lines indicate time, $t = 4M/\gamma$ for both cases, at which time 98.2% of the steady state velocity value has been obtained. The difference between the vertical lines is in the order of 1 hour.

al velocity in water and oil is shown.

2. Results and discussion

2.1 Application of the resistance force-velocity relationship for pore velocity transients

Consider now injection of fluids through porous media. The main assumptions used in the derivation are: (1) The inlet pressure increases sharply at time equal zero (Heaviside function) and remains constant for the rest of the time-period considered. Its magnitude is below the pressure difference which may induce turbulence, excluding flow in the Forchheimer regime. (2) The fluid and porous medium are incompressible. (3) The resistance or frictional force is proportional to the instantaneous average pore velocity, $u_P(t)$, and the friction coefficient is assumed constant independent of velocity. Since the level of description is phenomenological, the mean pore velocity is used, although a distribution of velocities is occurring during flow (Bijeljic et al., 2013; Berg and van Wunnik, 2017). (4) The porous medium is homogenous and inert to the fluid. Medium surface properties, i.e., the wettability may, however, impact the transient and will be briefly discussed later. (5) Steady state pore velocity is described by Darcy's law and is equal, u_P. Possible offset effects between pressure and pore velocity sometimes reported as deviations from Darcy's law are not considered (Bear, 1972). (6) The system is isothermal. (7) The size of the system is assumed "short", so the finite propagation of the pressure wave is neglected.

The relationship between the applied pressure difference acting on the fluid and the mean pore velocity, u_P, is under steady state conditions given by Darcy's law corrected for the porosity of the medium. Hence,

$$u_P = -\frac{K}{\phi\mu}\frac{\Delta P}{L} \tag{5}$$

Net force on the fluid is related to the difference in pressure between the outlet and inlet faces, $\Delta P = P_{\text{Out}} - P_{\text{In}}$, respectively, where $P_{\text{In}} > P_{\text{Out}}$. Both pressures are assumed constant from the start of fluid injection ($t = 0$) where the inlet pressure, P_{In}, is a step-function and remains constant for all subsequent times. P_{In} is delivered by a pump run in constant pressure mode. Eq. (5) can be written,

$$-\Delta P = \frac{\phi\mu L}{K}u_P \tag{6}$$

The instantaneous pressure reduction associated with the instantaneous resistance force is $F_R(t)/(A\phi)$, where $F_R(t)$ is the instantaneous flow resistance force and A is the cross-sectional area. $F_R(t)$ is time-dependent due to a time-dependent pore velocity, $u_P(t)$, and therefore given as,

$$F_R(t) = A\frac{\phi^2\mu L}{K}u_P(t) = \gamma_D u_P(t) \tag{7}$$

Eq. (7) is on the general, resistance force-velocity form, given by Eq. (2). Hence, the friction coefficient for porous media flow is, $\gamma_D = A\phi^2\mu L/K = \mu\phi V_{PV}/K$, where V_{PV} is the pore volume of the sample given as, $V_{PV} = V_B\phi = AL\phi$, where V_B is the bulk volume. Due to the assumptions used, the friction coefficient γ_D, can be considered constant independent of velocity. An expression for the time-dependent pore velocity during the transient build-up period can now be derived by considering a force balance on the fluid in the medium. Hence,

$$-\Delta PA\phi - \gamma_D u_P(t) = m_F\frac{du_P(t)}{dt} \tag{8}$$

where m_F is the mass of the fluid. The inertia term on the right-hand side is significant initially but decreases due to the increasing frictional force term and approaches zero at which time steady state pore velocity is established. Solving Eq. (8) for the pore velocity, $u_P(t)$, gives the same type of expression for the transient period as Eq. (4) for the falling object case. Hence,

$$\begin{aligned} u_P(t) &= -\frac{\Delta PA\phi}{\gamma_D}\left(1 - e^{-\frac{\gamma_D}{m_F}t}\right) \\ &= -\frac{K}{\phi\mu}\frac{\Delta P}{L}\left(1 - e^{-\frac{t}{T_R}}\right) \end{aligned} \tag{9}$$

The magnitude of the transient period is therefore specified by the relaxation term, $T_R = m_F/\gamma_D$, in the exponent. The friction coefficient follows from Eq. (7), $\gamma_D = \mu\phi V_{PV}/K$, and the mass of the fluid in the medium is, $m_F = \rho_F V_B\phi = \rho_F V_{PV}$, where ρ_F is the fluid density. Hence, the relaxation time, T_R, is given as,

$$T_R = \frac{m_F}{\gamma_D} = \frac{\rho_F V_{PV}}{\frac{\mu\phi V_{PV}}{K}} = \frac{\rho_F K}{\phi\mu} \tag{10}$$

The system relaxation time, T_R, is therefore characterized solely by intrinsic variables, i.e., variables which are independent of system size. Pore velocity as a function of time, $u_P(t)$, from time equal zero is then given from Eq. (9) as,

$$u_P(t) = -\frac{K}{\phi\mu}\frac{\Delta P}{L}\left(1 - e^{-\frac{t}{\frac{\rho_F K}{\phi\mu}}}\right) \tag{11}$$

Normalized pore velocity, $u_{pN}(t) = u_P(t)/u_P$, as a function of time can likewise be written,

$$u_{pN}(t) = 1 - e^{-\frac{t}{\frac{\rho_F K}{\phi\mu}}} \tag{12}$$

Table 1. Medium and fluid properties for the Basecase with medium values typical for naturally occurring subterrain reservoirs.

Quantity	Value
Water density, ρ_w	1000 kg/m^3
Water viscosity, μ	0.001 Pa·s
Medium fractional porosity, ϕ	0.20
Medium absolute permeability, K	9.87×10^{-13} m^2

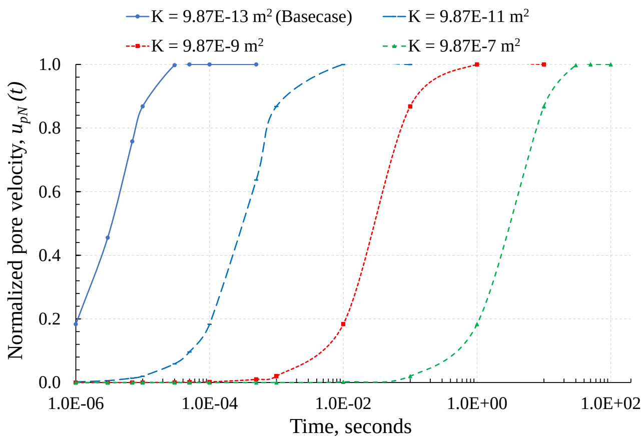

Fig. 2. The impact of varying permeability on the relaxation time to reach steady state pore velocity using Eq. (12) compared to Basecase values from Table 1.

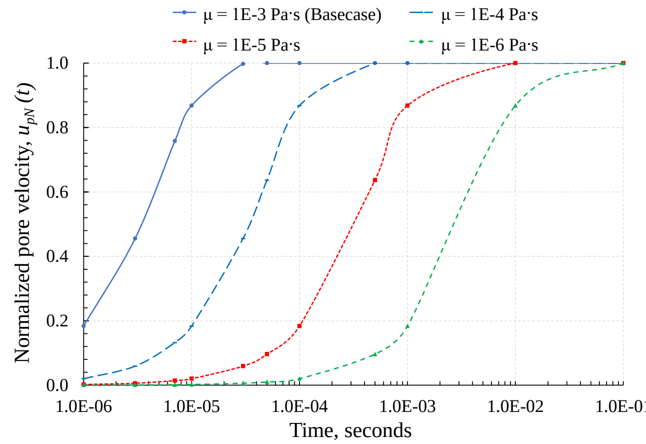

Fig. 3. The impact of varying fluid viscosity on the relaxation time to reach steady state pore velocity using Eq. (12) compared to Basecase values from Table 1.

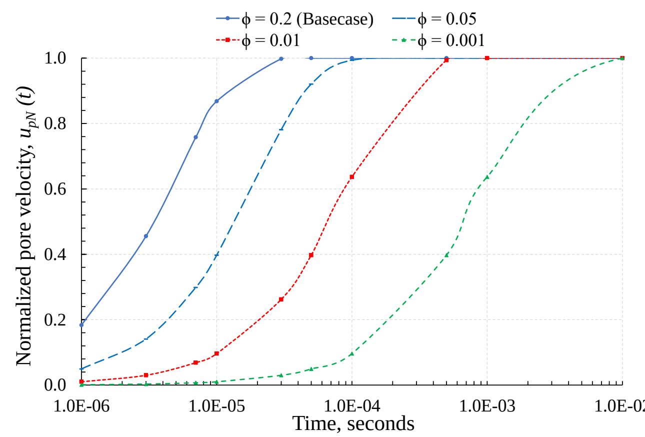

Fig. 4. The impact of varying medium fractional porosity on the relaxation time to reach steady state pore velocity using Eq. (12) compared to Basecase values from Table 1.

Eqs. (11) and (12) specify how fast the velocities, $u_P(t)$ and $u_{pN}(t)$, approach the steady state pore velocity value, u_P, described by Darcy's law. The quantities influencing the transient period are the fluid properties, viscosity and density, and the medium properties, porosity, and permeability. The transient period increases with increasing fluid density and medium permeability and with decreasing fluid viscosity and medium porosity. These predictions are all in line with expectations as increased fluid density and medium permeability causes larger inertia forces and less impact of frictional forces (permeability is proportional to fluid conductivity, i.e., the reciprocal of resistance), respectively. Likewise, decreasing fluid viscosity and medium porosity imply less damping and impact of frictional forces, respectively. As mentioned previously, steady state conditions may for all practical purposes be considered established when $t = 4T_R$. At this time, the transient pore velocity, $u_{pN}(t)$, has obtained 98.2% of the steady state terminal pore velocity value obtained from Darcy's law, u_P. Typical values and dimensions for laboratory medium samples when water is the injected fluid are given in Table 1, referred to as Basecase values. Calculated transient periods for variations in permeability, viscosity and porosity using Eq. (12) are illustrated in Figs. 2-4, respectively. It should finally be mentioned that the friction coefficient in Eq. (7) should be modified if energy calculations are to be performed. The reason is that the Darcy laws should be written, $-\Delta P = \mu Lq/(AK)$, to be completely analogous to Ohm's law. If $Z = \mu L/(AK)$ is defined as the resistance or impedance term, all the conventional energy expressions can be adapted from circuits theory valid for linear dissipative systems (Callen and Welton, 1951), i.e., the power dissipated will be, Power $= (-\Delta P)^2/Z = Zq^2 = -\Delta Pq$ (Smith, 1984). The friction factor in Eq. (7) is, however, used herein to emphasize the direct similarity between the movement of spheres falling through fluids and liquid flow through porous media cases.

Fig. 2 shows that the transient period increases with medium permeability and is in general negligible for typical values found for subterrain sandstone reservoir, typically in the range from 1 mD to 10 D. Extremely high permeabilities are required to see any significant relaxation time. The same trend can be seen with respect to fluid viscosity in Fig. 3. Even a viscosity value a factor 1,000 less than viscosity of water (1 mPa·s) at ambient conditions gives negligible transient periods. Fig. 4 confirms that the transient period does not become significant within normal porosity values either. A very small porosity value of 0.1% gives a transient period less than 0.1 s. Since permeability and porosity normally also are positively correlated, the analysis indicates that transient periods towards steady state pore velocity during laboratory injection tests should occur relatively rarely considering typical fluid and naturally porous media property values. All the transient value results presented in Figs. 2-4 are in the same range as the values obtained by Taherkhani and Pourafshary (2012) using similar input values when comparing propagation of pressure pulses through porous media using the conventional diffusion and telegrapher's equation, respectively.

The results presented therefore support that transient periods occasionally observed in laboratory flooding tests are not caused by inertia effects as they decay away very quickly for typical value ranges for fluid and porous media properties. If transient periods on the other hand, however, are observed, they are therefore originating from either more extreme fluid/medium property values than considered or other time-dependent fluid-medium interactions not accounted for by the relaxation time presented, e.g., fines migration (Rosenbrand et al., 2015), electrokinetic phenomena (Delgado et al., 2007), or other effects.

2.2 Discussion

The results support the applicability of the resistance force-velocity relationship to various transient challenges including fluid flow through porous media. Since the transient relates to the pore velocity rather than the pressure, the approach can in principle also be extended to account for velocity or shear-rate dependency if the fluid at hand is non-Newtonian. Polymers are typical examples of such fluids and used e.g., extensively for enhanced oil recovery purposes.

It is known that the properties of the medium surface will impact the local fluid flow profile. Hence, a surface wetted by the fluid may experience a slightly different transient compared to a non-wetting fluid which will have a considerable slip length and therefore flow more as a unit as no fluid molecules stick to the medium surface walls (see also below regarding the magnitude of the coefficient of restitution, ε). Whether such effects are measurable must be investigated experimentally.

The relaxation time for the basecase is in the order of 10^{-6} s. This value can be compared to other typical time scales in the system. On the microscopic level, the time scale between molecular collisions in liquid water is of the order of 10^{-12} s (Zwanzig, 1965). Time scales associated with the macroscopic description of viscous dissipation is characterized by the local velocity gradient, which typically gives momentum transfer fluxes in the range 1-100 s, see e.g., Berg and van Wunnik (2017). The values were estimated based on variation in individual pore scale fluxes for mean fluxes in the order of μm/s (0.3 m/day), typically occurring in laboratory fluid injection tests. Hence, the time scales for the inertia transients fall in between the molecular and the viscous dissipation time scales, in line with expectations.

The phenomenological approach used can also easily be extended if a more detailed description of the quantities impacting the permeability of the medium is required as discussed by Standnes (2022). A generalized form of Darcy's law was presented therein based on a more detailed description of the total frictional coefficient, which also accounts for thermal resistance in addition to the conventional viscous resistance. The former is in fact the flow resistance Knudsen flow generates when fluids driven by external pressure sources encounter objects in the flow path. It can be shown that Knudsen flow will occur in very low-permeable formations where the diameter of the pores approaches *nm* size containing gas of low density (Kuila et al., 2013; Lin et al., 2017). Hence, the mean free path of the molecules becomes larger than the pore diameter meaning that they will collide much more frequently with the pore walls than adjacent fluid molecules (Knudsen number, $K_n > 10$) (Kuila et al., 2013). Viscous resistance therefore disappears and Kuila et al. (2013) showed that the permeability, K_K, of the molecules in a tube with radius, r, in that limit can be expressed as,

$$K_K = \frac{2}{3}\frac{r\phi}{\tau}\sqrt{\frac{8M_W}{\pi RT}} \sim \frac{r\phi}{\tau}\sqrt{\frac{M_W}{RT}} \tag{13}$$

where τ is the tourtosity, M_W is the molar mass of the gas molecules, R is the gas constant and T is absolute temperature. K_K is basically the "self-conductivity" of the molecules accounting for mass transport along the tube caused by local density gradients i.e., gradients in the chemical potential (Kittel and Kroemer, 1980). Using Eqs. (12) and (13) in Standnes (2022), with $W = 0$, i.e., in absence of viscous resistance, thermal "conductivity" or permeability called K_T, which is equal to the reciprocal of thermal resistance (here expressed in seconds since the flux in Kuila et al. (2013) is mass flux), can be shown to equal (using the same quantities as in Eq. (13) for convenience),

$$K_T = \frac{3\pi\sqrt{2}}{16}\frac{L\phi}{1+\varepsilon}\frac{1}{\frac{SV_B}{2\widehat{A}}}\sqrt{\frac{M_W}{RT}} \sim \frac{L\phi}{1+\varepsilon}\frac{1}{\frac{SV_B}{2\widehat{A}}}\sqrt{\frac{M_W}{RT}} \tag{14}$$

Here ε is the coefficient of restitution during a gas-matrix collision ($\varepsilon = 1$ for elastic collisions), S and V_B are the specific surface area and bulk volume of the medium, respectively, and $\widehat{A}$ is a unit area. Hence, Eqs. (13) and (14) are similar in nature except that the radius r and the tortuosity factor τ in Eq. (13) have been replaced by the length of the medium L and the product of the factors, $(1+\varepsilon)$, and $SV_B/\left(2\widehat{A}\right)$, in Eq. (14), respectively. The similarity is expected since both expressions concerns the same physical phenomenon, i.e., exchange of energy and momentum in molecular-matrix collisions on the microscopic level. The differences are caused by the physical processes they aim to describe. As noted above, Knudsen flow in Eq. (13) describes the conductivity of free molecular flow along a tube. Eq. (14), however, is aimed to quantify the fluid conductivity under forced flow conditions when the fluid is moving through a porous medium with a total cross-sectional area, $SV_B/\left(2\widehat{A}\right)$, perpendicular to the macroscopic flow direction. K_T is therefore inversely proportional to the total surface area the molecules will experience upon propagating through the medium, represented by the term, $2\widehat{A}/(SV_B)$. Since Eq. (14) is derived from more fundamental principles, it can also account for the efficiency of the momentum transfer in the collisions by the $(1+\varepsilon)$ term. This shows that the thermal resistance term in fact represents and quantifies the flow resistance generated by

Knudsen flow, i.e., free molecular flow, when the flow is driven by external pressure sources. This is completely in line with the fluctuation-dissipation theorem (Callen and Welton, 1951; Kubo, 1966), i.e., molecular fluctuations responsible for the irregular Brownian motion phenomenon are also responsible for the flow resistance the Brownian particle experiences upon moving through the fluid. The grains in the porous medium are here interpreted as a collection of large Brownian particles and the effect of viscous resistance is excluded (Grassia, 2001). It was showed in Standnes (2018) that thermal resistance also is significant for flow through media having "conventional properties", because it amounted to approximately 25%-30% of the energy dissipated in conventional sandstone samples at ambient conditions. That fraction increased significantly with temperature since the thermal resistance increases with square root of temperature while the viscous resistance at the same time decreases almost exponentially with temperature for liquids. The two contributions became of equal magnitude at 60 °C already. It should finally be mentioned that thermal resistance is totally dominated when fluids are in relative motion to single macroscopic objects larger than nm size as demonstrated in Standnes (2021), relevant if a more detailed description of the resistance terms for the falling objects shown in Fig. 1 is required.

3. Conclusions

A new approach to determine the transient period towards steady state pore flow velocity for fluids propagating through porous media under constant pressure condition has been developed. The following conclusions are drawn:

- The methodology uses the general, resistance force-velocity relationship, conventionally used to determine transient build-up behavior of electric currents in circuits and for the velocity of falling objects experiencing liquid fluid resistance.
- The transient period caused by inertia effects is characterized by a relaxation time comprising fluid density and viscosity together with medium properties as porosity and absolute permeability.
- The period increases with decreasing porosity and fluid viscosity and with increasing fluid density and absolute permeability of the medium.
  - The transient period is negligibly small for typical fluid/medium property values characterizing typical subterrain sandstone reservoirs.
  - Observation of transient velocity instabilities in laboratory tests is therefore caused by other effects, e.g., fines migration and electrokinetic phenomena.

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Conflict of interest

The author declares no competing interest.

References

Alonso, M., Finn, E. J. Fundamental University Physics, 2nd Edition. Massachusetts, USA, Addison-Wesley Publishing Company, 1983.

Batchelor, G. K. An Introduction to Fluid Mechanics. Cambridge, UK, Cambridge University Press, 2000.

Bear, J. Dynamics of Fluids in Porous Media. North Chelmsford, USA, Courier Corporation, 1988.

Berg, S., van Wunnik, J. Shear rate determination from pore-scale flow fields. Transport Porous Media, 2017, 117(2): 229-246.

Bijeljic, B., Raeini, A., Mostaghimi, P., et al. Predictions of non-Fickian solute transport in different classes of porous media using direct simulation on pore-scale images. Physical Review E, 2013, 87: 013011.

Brace, W. F., Walsh, J. B., Frangos, W. T. Permeability of granite under high pressure. Journal of Geophysical Research, 1968, 73(6): 2225-2236.

Callen, H. B., Welton, T. A. Irreversibility and generalized noise. Physical Review, 1951, 83(1): 34-40.

Darcy, H. Les Fontaines Publiques de la Ville de Dijon, Dalmont. Paris, France, 1856.

Delgado, A. V., González-Caballero, F., Hunter, R. J., et al. Measurements and interpretation of electrokinetic phenomena. Journal of Colloid and Interface Science, 2007, 309: 194-224.

Dey, S., Zeeshan Ali, S., Padhi, E. Terminal fall velocity: The legacy of Stokes from the perspective of fluvial hydraulics. Proceedings of the Royal Society A, 2019, 475(2228): 20190277.

Dullien, F. A. L. Single phase flow through porous media and pore structure. The Chemical Engineering Journal, 1975, 10(1): 1-34.

Foster, W. R., McMillen, J. M., Odeh, A. S. The equations of motion of fluids in porous media: I. propagation velocity of pressure pulses. Society of Petroleum Engineers Journal, 1967, 7(4): 333-341.

Grassia, P. Dissipation, fluctuations, and conservation laws. American Journal of Physics, 2001, 69(2): 113-119.

Kittel, C., Kroemer, H. Thermal Physics. New York, USA, W. H. Freeman and Company, 1980.

Kubo, R. The fluctuation-dissipation theorem. Reports on Progress in Physics, 1966, 29(1): 255-284.

Kuila, U., Prasad, M., Kazemi, H. Assessing Knudsen flow in gas-flow models of shale reservoirs. Canadian Society of Exploration Geophysicists, 2013, 38: 23-27.

Landau, L. D., Lifshitz, E. M. Fluid Mechanics, Second Edition. Oxford, UK, Pergamon Press, 1987.

Lin, D., Wang, J., Yuan, B., et al. Review on gas flow and recovery in unconventional porous rocks. Advances in Geo-Energy Research, 2017, 1(1): 39-53.

Neuman, S. P. Theoretical derivation of Darcy's law. Acta Mechanica, 1977, 25: 153-170.

Odeh, A. S., McMillen, J. M. Pulse testing: Mathematical analysis and experimental verification. Society of Petroleum Engineers Journal, 1972, 12(5): 403-409.

Rosenbrand, E., Kjøller, C., Riis, J. F., et al. Different effects of temperature and salinity on permeability reduction by fines migration in Berea sandstone. Geothermics, 2015, 53: 225-235.

Slattery, J. C. Single-phase flow through porous media. AIChE Journal, 1969, 15(6): 866-872.

Smith, R. J. Circuits, Devices and Systems, 4th Edition. NewYork, USA, John Wiley & Sons, 1984.

Standnes, D. C. Implications of molecular thermal fluctuations on fluid flow in porous media and its relevance to absolute permeability. Energy & Fuels, 2018, 32: 8024-8039.

Standnes, D. C. Dissipation mechanisms for fluids and objects in relative motion described by the Navier-Stokes Equation. ACS Omega, 2021, 6(29): 18598-18609.

Standnes, D. C. Derivation of the conventional and a generalized form of Darcy's Law from the Langevin Equation. Transport in Porous Media, 2022, 141(1): 1-15.

Stokes, G. G. On the numerical calculation of a class of definite integrals and infinite series. Transactions of the Cambridge Philosophical Society, 1851, 9: 166-187.

Taherkhani, M., Pourafshary, P. Investigation of pressure pulse distribution in porous media. Journal of Chemical and Petroleum Engineering, 2012, 46(1): 41-52.

Whitaker, S. Flow in porous media I: A theoretical derivation of Darcy's Law. Transport in Porous Media, 1986, 1(1): 3-25.

Yang, D., Wang, W., Chen, W., et al. Revisiting the methods for gas permeability measurements in tight porous medium. Journal of Rock Mechanics and Geotechnical Engineering, 2019, 11: 263-276.

Zimmerman, R. W. Imperial College Lectures in Petroleum Engineering, The-Volume 5: Fluid Flow in Porous Media. Singapore, World Scientific, 2018.

Zwanzig, R. Time-correlation functions and transport coefficients in statistical mechanics. Annual Review of Physical Chemistry, 1965, 16(1): 67-102.