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Invited review

Geometrical, fractal and hydraulic properties of fractured reservoirs: A mini-review

Wei Wei*, Yuxuan Xia

Hubei Subsurface Multi-scale Imaging Key Laboratory, Institute of Geophysics and Geomatics, China University of Geosciences, Wuhan 430074, P. R. China

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Abstract: Fractures and fracture networks play an important role in fluid flow and transport properties of oil and gas reservoirs. Accurate estimation of geometrical characteristics of fracture networks and their hydraulic properties are two key research directions in the fields of fluids flow in fractured porous media. Recent works focusing on the geometrical, fractal and hydraulic properties of fractured reservoirs are reviewed and summarized in this mini-review. The effects of several important parameters that significantly influences hydraulic properties are specifically discussed and analyzed, including fracture length distribution, aperture distribution, boundary stress and anisotropy. The methods for predicting fractal dimension of fractures and models for fracture networks and fractured porous media based on fractal-based approaches are addressed. Some comments and suggestions are also given on the future research directions and fractal fracture networks as well as fractured porous media.

Keywords: Fracture, fractal, hydraulic properties, permeability.

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1. Introduction

Flow and transport in fractured systems takes a significant role in geosciences and geoengineering, such as enhanced oil recovery, CO_2 sequestration and geothermal energy development (Berkowitz, 2002; Gerritsen and Durlofsky, 2005; Juanes et al., 2006; MacMinn et al., 2010; Haugen et al., 2012; Mora et al., 2015). For tight oil and gas reservoirs, the fracture permeability is much larger than that of matrix. As a result, the permeability of matrix can be negligible and fluid flow in fractures is supposed to follow the classical cubic law. During the past several decades, many models and methods have been devoted to the permeability of fractured porous media. However, the geometrical and hydraulic properties have not been clearly understood due to the complex morphologies of both single fracture and fracture networks.

There are generally three conceptual models to simulate fluid flow and transport in fractured porous media, including stochastic continuum model, channel network model and discrete fracture network model (Selroos et al., 2002; Figueiredo et al., 2016). The stochastic continuum model can represent the fractured systems as equivalent homogenous media, in which the anisotropy of permeability cannot be considered (Tsang et al., 1996). The channel network model can simulate flow properties of fractures that are tortuous and exhibit preferential pathways, which is more realistic than the stochastic continuum model (Neretnieks, 1987; Figueiredo et al., 2016). However, when the aperture heterogeneity of each fracture in complex fracture networks is considered, it is a time-consuming and even unavailable task for calculations. The discrete fracture network model that neglects the influence of rock matrix permeability has been rapidly developed during the past 40 years (Min and Jing, 2003; Liu et al., 2016b). From Fig. 1, there are many factors that influences the permeability of a discrete fracture network such as fracture persistence, orientation, surface roughness, aperture, filling, location, wall strength (Liu et al., 2016a, 2016c). However, the fractures are usually invisible because they are buried in the deep underground and it is quite difficult to characterize these geometric properties. Fortunately, these geometric parameters such as fracture surface roughness, fracture length and aperture are fractals and fractal technology has been successfully applied on transport properties of fractures.

The present study aims to review the available works on geometrical, fractal, and hydraulic properties of fracture reser-

*Corresponding authors. E-mail: weiwei@cug.edu.cn

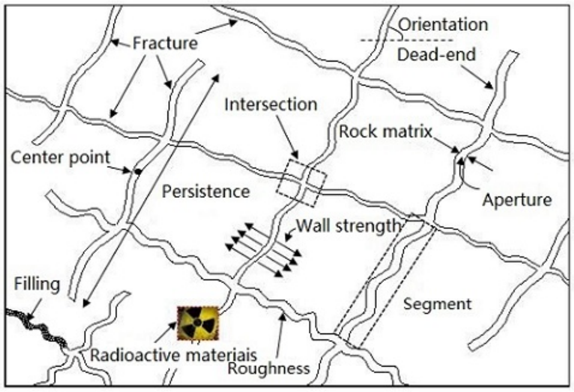

Fig. 1. Schematic view of fractured porous media (Liu et al., 2016b).

voirs. Section 2 introduces the relationships between geometric properties and permeability of fracture networks. Section 3 exhibits the permeability estimation using the fractal method, and Section 4 discussed the remaining problems in quantifying the fractal and hydraulic properties of fracture networks. This work can provide a framework to understand the previous achievements and their possible topics in future research.

2. Geometric properties of fracture networks

Generally, flow properties of a natural porous media containing fracture network is highly dependent on the interconnection degree of fractures and the heterogeneity of individual fracture characters, in which the matrix permeability is assumed to be low enough to be neglected (Long and Witherspoon, 1985). The permeability of fracture networks is influenced by their geometric characterizations, such as fracture length distribution, aperture distribution, Boundary stress and anisotropy. The length and aperture distributions of fractures are two main parameters for the heterogeneity of fractured system. Geometric properties and permeability of fracture networks are briefly analyzed and discussed in this section.

2.1 Fracture length distribution

Based on field survey on discontinuities of rock masses, the fracture lengths have been found to obey exponential, power-law, and lognormal distributions (Chang and Yortsos, 1990; Chelidze and Gueguen, 1990; Andrade Jr et al., 2009; Kolyukhin and Torabi, 2013).

Many numerical models fail to consider the random geometry of natural fracture systems. Considering the problem of defining fracture permeability and elasticity tensors for each grid block in a fractured material, Brown and Bruhn (1998) developed an improved fractured reservoir model, in which the permeability tensor is related to the crack geometry. Long and Witherspoon (1985) numerically investigated how interconnection degree affects the magnitude and nature of fracture permeability. The interconnection of fractures is decided by fracture density and fracture size. Their results show that the degree of interconnection increases with the increased of fracture length. Using a power law model of length distribution and a lognormal model for aperture distribution, De Dreuzy et al. (2001) studies the influence of fracture length and aperture distributions on the permeability of bidimensional synthetic fracture networks, and analyzed two endmost models which fracture aperture and length are independent and perfectly positively correlated.

Leung and Zimmerman (2012) generated discrete fracture network models where fracture lengths following the power-law distribution, and linked the fracture permeability to the fracture geometric parameters. Their approach is capable of estimating the fracture permeability K that spans over ten orders of magnitude.

$$K = K_0 B \sqrt{1+2\zeta} \frac{N l_M}{2L} \tag{1}$$

where K_0 is fracture permeability consisting of an orthogonal pair of fractures, each passing straight through the system, B is dimensionless constant, N is fracture number, l_M is the arithmetic mean of all fracture lengths, and ζ is fracture connectivity.

For the effect of fracture height on enhancing gas recovery, Cai and Sun (2013) developed an analytical model for characterizing spontaneous imbibition horizontally from a single plane fracture into gas-saturated matrix blocks with gravity force included in the entire imbibition process, and accordingly analyzed the mechanism of fracture-enhanced spontaneous imbibition. Cai et al. (2013) further analyzed the criterion (characterized by inverse bond number) for co-current or counter-current manner of spontaneous imbibition fracture-matrix dual porosity medium, and proposed a fractal model for spontaneous imbibition mechanism of dual-porosity medium based on the fractal characteristics of pores in porous matrix.

2.2 Aperture distribution

The hydraulic aperture of single fractures has been extensively studied during the previous studies, which is significantly related with fracture surface roughness (Olsson and Barton, 2001), Reynolds number (Zimmerman and Main, 2004; Xiong et al., 2011), contact (Zimmerman and Bodvarsson, 1996; Li et al., 2008), shear process (Javadi et al., 2014), hydraulic gradient (Guha Roy and Singh, 2015), etc. Besides, in-site observations show that fracture aperture is also correlated with fracture length, following power law functions with the exponent in the range of 0.5-2.0 (Fossen and Hesthammer, 1997). However, only a few works focused on the distribution of apertures in 2-D and/or 3-D discrete fracture network models.

By assuming that the fracture aperture follows a lognormal distribution, De Dreuzy et al. (2001; 2002) studied the hydraulic flow of 2-D random fracture networks, and established a mathematical expression for network permeability. However, the lognormal distribution of fracture aperture shows a long tail with a small amount but extremely large values. The distribution of this long tail exhibits random properties and can change fracture permeability. Baghbanan and Jing (2007) used a truncation threshold to decrease the sampling bias with apertures and developed an formula to relate fracture aperture to fracture length:

$$g'(e) = g'(e_a) + [g'(e_b) - g'(e_a)] \frac{l^{-D_f} - l_{min}^{-D_f}}{l_{max}^{-D_f} - l_{min}^{-D_f}} \tag{2}$$

where g' is error function, and e_a and e_b are the lower and upper aperture limits, respectively. The results show that fracture permeability decreases when applying a small stress ratio of horizontal stress to vertical stress.

2.3 Boundary stress

The applied boundary stress can decrease the aperture of fractures and consequently decrease the fracture permeability (Roman et al., 2012; Zhao, 2013; Indraratna et al., 2015). To estimate the influence of stress on fracture permeability containing one or multiple sets of oriented fractures, Chen et al. (2007) proposed an equivalent model to calculate strain-dependent permeability of fractured rock.

Using this method, the hydraulic properties of a single fracture subjected to normal and shear stresses can be characterized using a closed-form solution. To illustrate the coupling effect of fluid flow and stress/deformation, Zhou et al. (2008) derived an analytical model to calculate the coupled flow-stress permeability tensor based on the superposition principle of liquid dissipation energy. This model considers the effects of pre-peak shear dilation and shear contraction, and can predict the magnitude of fracture network permeability.

Considering the influence of coupled shear-flow, increasing the number of closure fracture results in the flow rate of single fractures decreasing, and the variety is nonlinearly with the increment of boundary stress (Pyrak-Nolte and Morris, 2000). Baghbanan and Jing (2008) derived a function in terms of the normal stiffness of each fracture and normal stress, as

$$k_n = \frac{(10\sigma_n + \sigma_{nc})^2}{9\sigma_{nc} e_i} \tag{3}$$

where k_n is normal stiffness, σ_n is normal stress, σ_{nc} is critical normal stress, and e_i is aperture of the ith fracture.

2.4 Anisotropy

Due to the existence of discontinuities in porous rocks, both fractured rock masses and permeability exhibit anisotropic properties (Vu et al., 2013; Barton and Quadros, 2015). Chen et al. (1999) proposed an analytical model to predict the permeability tensor of fractures nested in parallel-plate type configurations. For a fracture network containing multiple sets of fractures, the permeability tensor (k_{ij}) can be obtained as follows:

$$k_{ij} = \frac{1}{12} \sum_{m=1}^{M} \phi_i^{(m)} [e^{(m)}]^2 \Omega_{ij}^{(m)} \tag{4}$$

where ϕ_i is equivalent porosity in i-direction, $e^{(m)}$ is the mth fracture aperture, $\Omega^{(m)}$ are the mth fracture coordinate transformation coefficients, and M is the total number of fracture sets.

Liu et al. (2016d) performed a numerical study to analyze the directional permeability of fracture networks containing two fracture sets, in which the fracture length distribution follows the fractal scaling law proposed in their early works (Liu et al., 2015). With increasing the ratio of fractal dimensions of fractures, the direction of the maximum permeability moves towards the fracture set with a larger fractal dimension. When the ratio of apertures of the two fracture sets increases from 1 to 5, the ratio of the maximum and minimum permeability decreases from 9.93 to 4.55, whereas this ratio increases from 2.79 to 5.35 for intersection angles of the two fracture sets varying from 30 to 60.

A large number of factors, such as fracture density, length, aperture, orientation, and model scale, influence the anisotropic properties of fracture permeability (Liu et al., 2016c). To characterize anisotropic hydraulic behaviors of fracture networks, it is necessary and important to develop a generic mathematical expression by considering these factors synthetically.

3. Fractal-based permeability of fracture networks

3.1 Fractal properties of fracture networks

Besides the fractal scale characters of pores and streamlines in natural porous media, fractures and fracture networks in porous media are also verified to be fractals at micro-structural and the macro-structural levels (Babadagli, 2001; Askari and Ahmadi, 2007; Sahimi, 2011; Miao et al., 2015a). Okubo and Aki (1987) reported that the fractal dimensions are in the range of 1.12-1.43 for the San Andreas Fault based on the fractures in a 30 km wide range. Vignes-Adler et al. (1991) presented that fracture fractal dimension is in the range of 1.4-1.5 for fractures obtained from images at different scales. Velde et al. (1991) found that the fracture networks are fractal at several length scales. Barton and Zoback (1992) analyzed the trace lengths of fractures spanning ten orders, ranging from micro to large fractures and obtained the fractal dimension of fractures in the range of 1.3-1.7. Barton (1995) calculated the fractal dimensions of 17 fracture networks at Yucca Mountain, Nevada. The results show that the fractal dimension ranges from 1.3 to 1.7. Liu et al. (2015) simulated fluid flow in fractal-based rock fracture networks with the fractal dimension of 1.3-1.6. Miao et al. (2015b) derived the analytical expressions for the permeability of fracture networks based on the fractal length distribution of fractures. More advances on fractal properties of fracture systems can consult to the recent review works (Kruhl, 2013; Liu et al., 2016c).

Generally, the stress concentration zone extent strongly impacts the fractal dimension of discrete fracture network models from of the study by Bonneau et al. (2016). It seems difficult to constrain precisely the expected correlation dimension according to natural network characterization. It is found that, from Liu et al. (2016d), with the increment of the mean fractal dimension of fractures, the side length of discrete fracture network models at the REV size decreases with an exponential function, which could be utilized to predict the REV size of a fracture network through calculating its fractal dimensions. These examples indicate that discrete fracture network model can be described with fractal theory.

3.2 Methods for measuring the fractal dimension

The Box-counting method is the most popular technique to decide fractal dimension, which is a method of data collection for analyzing complex patterns by subdividing a dataset or image into smaller pieces and scale. The essence of the process lies in examining how observations of detail change with corresponding scale. When using the Box-counting method, the researchers change the size of the element to check the fractal behavior of the object extracted from digital media. Specifically, mapped fracture system is superimposed using square box of size r, and the number of the occupied boxes covering the object $N(r)$ is calculated by

$$N(r) \propto r^{-D} \tag{5}$$

From Eq. (5), the fractal dimension D can be obtained by linearly fitting the data points in log-log space ($N(r)$ versus r) (Fig. 2). A variety of scanning strategies has been used in Box-counting algorithms. Computer based Box-counting algorithms which usually are compiled through C/C ++, Matlab language, have been applied to patterns in 1-, 2-, and 3-D spaces (Xie et al., 2010; Ge et al., 2015). The object or digital image is usually cropped to a size of 2^N to obtain the continuously bisected dataset. Detailed algorithm steps can refer to the recent publication (Wang et al., 2012). In fracture networks, a linear feature, an in-depth review has been presented by Klinkenberg (1994).

However, numerous variations of Box-counting technique such as box flex method and box rotate method, and other versions of this method usually cannot overcome the influence of the minimum scale/cell size, which may result in the fractal dimension in 2D slice lager than 2 (Li et al., 2009). Considering the limitation of Eq. (5) by the lower limiting value, Roy et al. (2007) developed a new box-counting algorithm to predict the fractal dimension of fracture network, in which the minimum box size r_{min} is replaced by a proxy value r_{cmin}. Fitting data $\log N(r)$ versus $\log r$ by least square method, the standard deviation of the slope s can be obtained and written as

$$s = \sqrt{\frac{\sum_{i=1}^{n}(y_i - y_{ai})^2}{(c-2) \mid \sum_{i=1}^{n} x_i^2}} \tag{6}$$

where y_i is observed $\log N(r)$, y_{ai} is predicted $logN(r)$, x_i = $\log(r)$ and c is the number of observations. When the variable ds/dr approaches to 0, the cutoff value of box size r_{cmin} can be identified. Then, the fractal dimension can be calculated by the data between r_{cmin} and r_{max}.

The fractal dimension of fracture network extension of soil was calculated by Cai et al. (2017), from Fig. 3. Although these two methods have good fitting correlation coefficients (> 0.99), their standard deviations obtained from the improved Box-counting technique are lower an order of magnitude than that from the classical Box-counting method.

3.3 Relationships between fractal dimension and permeability

Since the fractal characters of fracture networks, its flow properties is usually related to the fractal dimension of fractures by numerical and analytical methods. Tree-like branched geometry played a unique role and has steadily attracted many researchers in recent years in variety of fields. In low permeability reservoirs, fractures induced by hydraulic actions form a complex multiple fracture network. Tree-like branched network models and assumptions are used to analyze their transport and flow properties of fractured reservoirs and porous media embedded with tree networks (Li et al., 2016a; Li et al., 2016b; Fan and Ettehadtavakkol, 2017).

Xu et al. (2006) analyzed the hydraulic conductivity and developed a permeability models of fractal-like tree networks between one point and a straight line, and also derive an analytical expression for the effective permeability for the network based on the parallel and series models. The effective permeability of a fractal-like tree network is expressed as:

$$K = \frac{d_0^2}{32}[\frac{1-N^{-1/D_l}}{1-N^{-(m+1)/D_l}} \frac{1-N^{(4/D_d-1/D_l-1)}}{1-N^{(4/D_d-1/D_l-1)(m+1)}}]^{1/2}$$

$$[1+\frac{N^{-1/D_l}(1-N^{-m/D_l})}{1-N^{-1/D_l}}\cos\theta] \tag{7}$$

where d_0 is the diameter of the 0^{th} branching level, N is the branching number, D_l is the fractal dimension of channel length distribution, and D_d is the fractal dimension for diameter distribution, m is total branching level. Based on Eq. (7), the influences of geometry structures of the network on effective permeability can be analyzed. Based on fractal and constructal tree networks, Xu et al. (2008) also presented an analysis of the radial flow in heterogeneous porous media, which is simulate by a dual-domain model based on the fractal distribution of pores and tortuosity of streamlines. They obtained analytical expressions for seepage velocity, pressure drop, local and global permeability of network and binary system, and discussed the transport properties of the optimal branching structure.

Recently, Miao et al. (2015b) developed a fractal scaling law for length distribution of fractures and a relationship between fractal dimension for fracture length and fracture area porosity. They also proposed a fractal permeability model for fractured rocks based on the fractal geometry and the cubic law for laminar flow in fractures.

Based on outcrop measurements of geothermal reservoirs in southwestern Turkey, Jafari and Babadagli (2012, 2013) found that the fracture length distribution exhibits fractal properties, and then established two functions to calculate fracture network permeability.

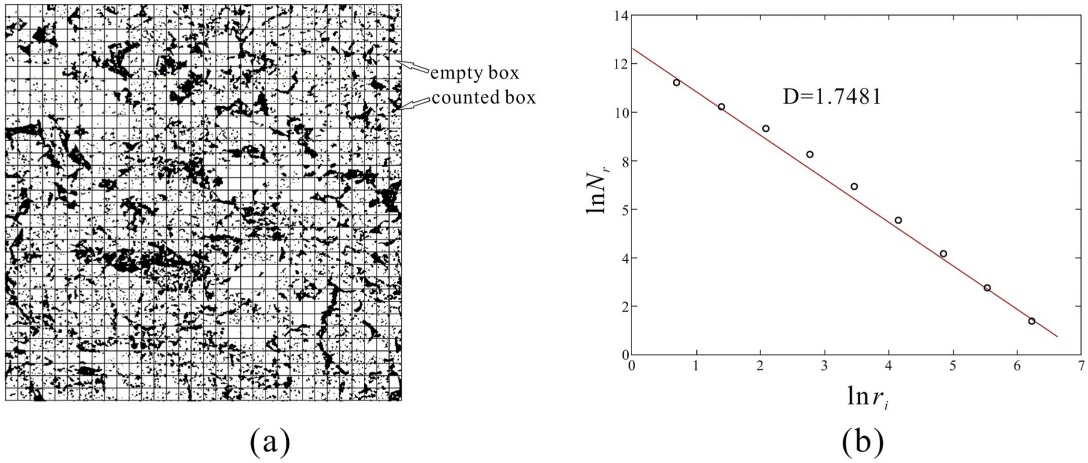

Fig. 2. Box-counting dimension results for digital image of sandstone with 1024^2 pixel (Chen et al., 2017). (a) Grid division schematic with the box size 32×32, (b) Double-log plot of $N(r)$ versus r_i with the slope equal to 1.7481, namely D = 1.7481.

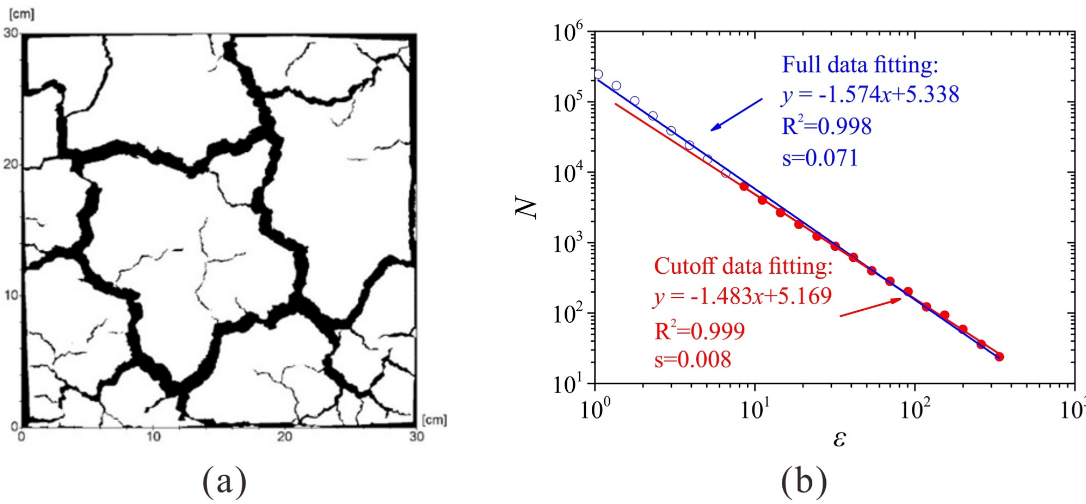

Fig. 3. The fractal dimension determination by Roy's method, (a) The original fracture network extension of soil. (b) Fractal dimension of (a), including the whole range of data points (s = 0.071) and those for which r > r_{cmin} were "turned off" (s = 0.008) (Cai et al., 2017).

The validity of these expressions has been verified by applying them to different fracture patterns that span a wide range of length values.

Liu et al. (2015) proposed a fractal equivalent permeability model for randomly distributed fracture networks, in which the fracture length follows a fractal distribution. Their numerical simulation results show an exponential relationship with the fractal dimension, D_f, as:

$$K = 2\times 10^{-13}\cdot \exp\left(0.9784D_f\right) \tag{8}$$

Due to the influence of random numbers utilized for generating fracture lengths, orientations, and locations, from Eq. (8), the equivalent permeability may vary from five orders of magnitude when D_f = 1.3 to less than two orders of magnitude when D_f = 1.8.

4. Discussions and future works

Although a variety of subjects have been studied related to geometrical, fractal and hydraulic properties of fractured porous media such as rock masses and reservoirs, a gap still exists between theoretical knowledge and field practice. It is need to seek new theoretical and numerical studies and advances in various subjects addressing flow and transport mechanism as well as hydrocarbon recovery improvement, such as innovative stimulation techniques, reservoir characterization, and other approaches.

Specifically, not all the length distribution of fractures and fracture networks are follows the fractal law. They may be multi-fractal, and even non-fractal. Thus, more elaborate explorations are need for adequately characterizing the complex fractured networks. As we discussed in above section, fractal dimension is one of most important parameters to quantitatively characterize the complexity of fractures. However, fractal dimension is sensitive to prediction methods, even some irrational values may be obtained (Roy et al., 2007; Cai et al., 2017).

Future works also should be focused on the influence of fracture surface roughness, hydraulic gradient, the coupled thermo-hydro-mechanical-chemical processes

(Berkowitz, 2002; Liang et al., 2015; Yasuhara et al., 2016), precipitation-dissolution and biogeochemistry (MacQuarrie and Mayer, 2005), and so on, on the evolution of fracture network permeability.

5. Conclusions

Geometrical properties of fractured porous media is vital to predict and evaluate the hydraulic transport properties of fracture networks. This work briefly summarized and reviewed some subjects related to geometrical, fractal and hydraulic properties of fractured reservoirs, especially for the fractal approach and the theoretical expressions of permeability for fractured porous media by means of fractal geometry technology. Some future directions and works are anticipated about fracture networks on fractured porous media.

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