


## Original article

# Stress dependent gas-water relative permeability in gas hydrates: A theoretical model

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### Abstract:

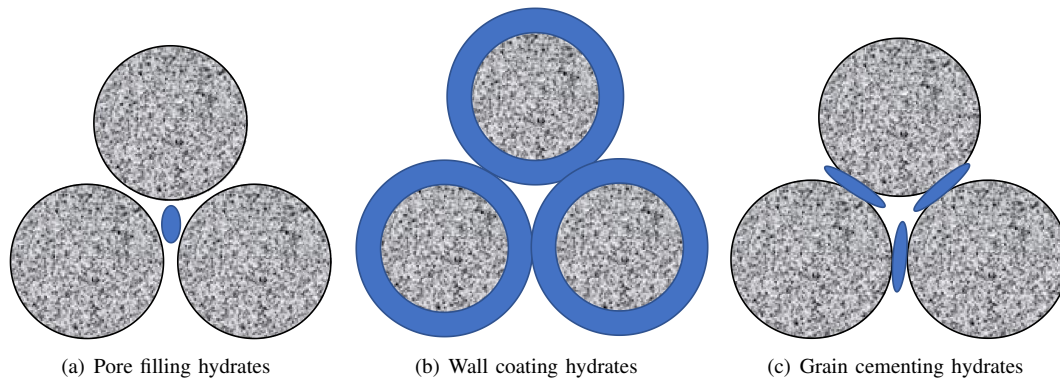
Research activities are currently being conducted to study multiphase flow in hydrate-bearing sediments (HBS). In this study, in view of the assumption that hydrates are evenly distributed in HBS with two major hydrate-growth patterns, i.e., pore filling hydrates (PF hydrates), wall coating hydrates (WC hydrates) and a combination of the two, a theoretical relative permeability model is proposed for gas-water flow through HBS. Besides, in this proposed model, the change in pore structure (e.g., pore radius) of HBS due to effective stress is taken into account. Then, model validation is performed by comparing the predicted results from the derived model with that from the existing model and test data. By setting the value of hydrate saturation to zero, our derived model can be reducible to the existing model, which demonstrates that the existing model is a special case of our model. The results reveal that, under the same saturation, relative permeability to water  $K_{rw}$  (or gas  $K_{rg}$ ) in PF hydrates is smaller than that in WC hydrates. Moreover, the morphological characteristics of relative permeability curve (relative permeability versus gas saturation) for WC hydrate and PF hydrate are different.

## 1. Introduction

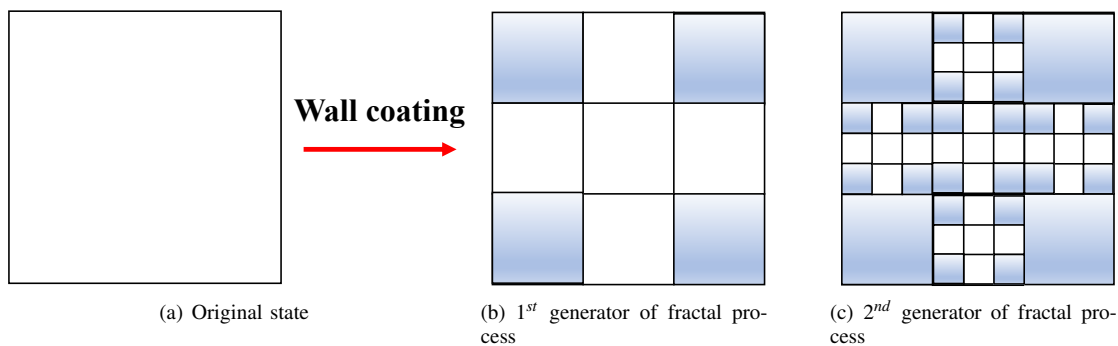
Over the past few decades, gas production from hydrate-bearing sediments (HBS) has already become the focus studied and front field, and demonstrated the good development prospect. However, until now, gas hydrates have not been successfully produced (Stoll and Bryan, 1979; Aya et al., 1997; Kvenvolden, 1998a, 1998b; Sloan, 1998; Dickens, 2003; Tajima et al., 2004; Lee et al., 2010; Cui et al., 2018; Sun et al., 2018).

As suggested by previous scholars (Yousif et al., 1991; Minagawa et al., 2009; Ordóñez et al., 2009; Waite et al., 2009; Kumar et al., 2010; Johnson et al., 2011; Liang et al., 2011; Dai et al., 2012; Li et al., 2014; Joseph et al., 2016), many physical properties of porous media will be strongly influenced by hydrates, which can be categorized as pore filling hydrates (PF hydrates), wall coating hydrates (WC hydrates), grain cementing hydrates (GC hydrates), and other types of

hydrates (e.g., discrete nodules, lenses, or veins) (seen in Fig. 1). Helgerud (2001) concluded that, PF hydrates nucleation occurred in the pore bodies (Fig. (1a)), WC hydrates formed on the particle surface (Fig. (1b)), GC hydrates grew at the contacts of grains (Fig. 1(c)) and other types of hydrates formed and grew randomly. Generally speaking, the actual growth type of hydrates is complex. Waite et al. (2009) and Zheng et al. (2018) suggested that hydrate-growth patterns were affected by various factors (e.g., hydrate saturation, rock petrophysical parameters, stress history, etc.). Specifically, Berge et al. (1999) and Delli and Grozic (2013, 2014) stated that, gas hydrates might change from WC hydrate to PF hydrate as hydrate saturation increased. To capture the main features of hydrates morphology in hydrate-bearing sediments (HBS) and simplify the model, many scholars (Waite et al., 2009; Singh, 2017; Singh et al., 2019a, 2019b) supposed that only PF hydrates, WC hydrates, and a combination of the two existed in the HBS. Previous studies reveal that the interspaces



**Fig. 1.** Different hydrate-growth patterns in porous media: (a) PF hydrates; (b) WC hydrates; (c) GC hydrates. The grey part is rock particles, and the blue part is gas hydrates.



**Fig. 2.** Fractal structure of HBS with WC hydrates: (a) the 1<sup>st</sup> generator of fractal process for WC; (b) the 2<sup>nd</sup> generator of fractal process for WC. The white part is void, and the blue part is solid.

in gas hydrate sediments possess fractal characteristics (Ji et al., 1997; Fini et al., 2001; Zhao et al., 2002; Zhang et al., 2006; Kirchmeyer et al., 2015). Taking fractal WC hydrates process as an example, assuming that HBS is composed of pores and solids, a fractal HBS can be generated (Fig. 2) with the wall-coating process being repeated at increasingly small scales. As depicted in Fig. 2(a), the original state of void is simplified as a plane (Ji et al., 1997). For every fractal process of WC (Figs. 2(b) and 2(c)), the void will be divided into 9 smaller parts and 4 corner small parts will be occupied by gas hydrates.

The production potential of gas hydrates mainly depends on permeability characteristics of the bearing sediments as high permeability could promote the production rate. Furthermore, due to the gas hydrates dissociation, multi-phase flow in HBS will occur, makes it important to determine relative permeability in HBS (Zheng et al., 2018). In addition, the permeability is affected by the geomechanical conditions, e.g., effective pressure, which varies during depressurization that is one of the commonly proposed techniques in dissociating gas hydrate (Daigle, 2016). Therefore, an accurate characterization of relative permeability behavior in gas hydrate reservoirs under stress conditions has significance for sustainable gas production in deformable HBS (Nimblett and Ruppel, 2003; Waite et al., 2009; Terzariol et al., 2017; Yang et al., 2018).

Experimental determination of permeability requires the retrieval of hydrate bearing samples undisturbed, which is a

challenging task. Moreover, it necessitates a precise control of pressure and temperature to ensure the maintenance of constant thermodynamic and geomechanical conditions, which is usually costly expensive. In addition, the obtained permeability is only applicable to specific hydrate-bearing medium, which becomes a cumbersome limitation (Yousif et al., 1991; Minagawa et al., 2009; Ordonez et al., 2009; Kumar et al., 2010; Johnson et al., 2011; Liang et al., 2011; Li et al., 2014; Joseph et al., 2016).

As an attractive alternative, theoretical methods for modeling and simulating relative permeability in HBS have also been developed (Wang et al., 2015; Kang et al., 2016; Mahabadi et al., 2016a, 2016b). The curve-fitting method is straightforward but contains empirical constants, whose physical meanings are not clear (Masuda et al., 1997). Kleinberg et al. (2003) derived a parallel capillary model and Kozeny grain model to study relative permeability of HBS containing two types of hydrates, i.e., PF and WC hydrates. Physically, PF hydrates nucleation occurs in the pore bodies, and WC hydrates form on the particle surface and grow gradually deep into the interior region of pores. Daigle (2016) proposed a permeability model for fractal HBS based on percolation theory. Singh et al. (2019a, 2019b) derived analytical models to investigate relative permeability in hydrate-gas-water systems with an evenly distributed pore radius. Recently, Liu et al. (2019, 2020) used fractal theory to describe pore structure of HBS, and derived fractal-based models to study permeability and relative

permeability in HBS. Moreover, Zhang et al. (2020) derived a fractal-based water permeability reduction model to explain the saturated water permeability reduction in HBS during hydrate formation. However, these models did not consider effective stress, which has been shown to influence the properties of HBS significantly (Lele et al., 2012, 2013; Liu et al., 2016). Therefore, it is significant to derive analytic methods for stress-dependent relative permeability of deformable HBS. Physically, HBS can be regarded as unsaturated porous media as shown in Fig. 1, and the strategy of research follows the transport in unsaturated porous media. In this paper, fluid flow in HBS is regarded as fluid flow in unsaturated porous media, and we derive a theoretical model for predicting stress-dependent relative permeability in HBS with the assumption that the pore structures of gas hydrate sediments under stress conditions are statistically self-similar fractals. Compared with the previous models (Daigle, 2016; Liu et al., 2019, 2020; Singh et al., 2019a, 2019b; Zhang et al., 2020), our proposed model takes effective stress into account.

In this paper, firstly, a stress dependent relative permeability model of HBS will be established. Subsequently, the derived model will be validated using the test results. Finally, the influences of relevant parameters on relative permeability will be evaluated.

## 2. Methodology

In this section, the analytical gas-water relative permeability model for HBS is presented. The interspaces in HBS are described using fractal geometry theory. Due to effective stress, pore radius of HBS will be narrowed, which can be quantitatively characterized with Hertz theory. More details about stress dependent pore radius and stress dependent permeability of porous media can be found in the former literature (Lei et al., 2019a, 2019b). For gas-water flow in the HBS, relative permeability will be affected by hydrate-growth patterns. Specifically, we developed a relative permeability model with the assumption that hydrates are uniformly distributed in cylindrical pores of fractal porous media with two major hydrate-growth patterns, i.e., PF hydrates, WC hydrates, and a combination of the two. In addition, for a given capillary, capillary radius does not change along the flow direction. The analytical model is derived under the following assumptions:

1. A pore with equivalent radius smaller than the critical value  $r_{c0}$  is occupied by wetting phase fluid (e.g., water phase), while this part of water cannot move and is regarded as residual water. It's supposed that in this paper residual water will not change into hydrate.

2. With regard to a capillary with a pore radius larger than  $r_{c0}$ , three phases (i.e., hydrate, gas and water) are evenly distributed in the HBS with different hydrate-growth patterns (WC hydrates, PF hydrates, and a combination of these two patterns) (Fig. 3). The rock wettability of HBS is water wet, and gas is considered as the non-wetting phase. There are no interactions between different phases.

3. With the pressure drop  $\Delta p$ , the steady-state, isothermal flow in pores with equivalent radius larger than  $r_{c0}$  is the continuum scale flow. Especially, gas-water two-phase flow

in pores with pore radii larger than  $r_{c0}$  can be simulated by solving continuous Navier-Stokes equations with the effects of gravity and buoyancy being neglected.

In view of the assumptions above, the residual water saturation  $S_{wc}$  in the capillary with a pore radius smaller than  $r_{c0}$  is (Xu et al., 2013; Lei et al., 2017):

$$S_{wc} = \frac{\int_{r_{\min}}^{r_{c0}} f(r) \pi r^2 L_{\tau} dr}{\int_{r_{\min}}^{r_{\max}} f(r) \pi r^2 L_{\tau} dr} = \frac{r_{c0}^{3-D_T-D_f} - r_{\min}^{3-D_T-D_f}}{r_{\max}^{3-D_T-D_f} - r_{\min}^{3-D_T-D_f}} \quad (1)$$

where  $r$  is pore radius;  $f$  is pore size distribution;  $L_{\tau}$  is the length of tortuous capillary;  $D_f$  is the pore fractal dimension and  $D_T$  is the tortuosity fractal dimension. In addition, the subscript max represents the maximum; and min represents the minimum. Mathematically, based on fractal theory, we have (Yu and Li, 2001; Yu and Cheng, 2002; Lei et al., 2019a, 2019b)

$$\left\{ \begin{array}{l} D_f = 2 + \frac{(D_{fn} - 2)r_{\max n}}{(3 - D_{fn})r_{\max} + (D_{fn} - 2)r_{\max n}} \\ D_T = 1 + \frac{\ln \left\{ \frac{1}{2} \left[ 1 + \frac{1}{2} \sqrt{1 - \varphi} + \frac{\sqrt{(1 - \sqrt{1 - \varphi})^2 + \frac{1}{4}(1 - \varphi)}}{1 - \sqrt{1 - \varphi}} \right] \right\}}{\ln \left( \frac{D_f - 1}{\sqrt{D_f}} \sqrt{\frac{1 - \varphi}{D_f}} \frac{\pi}{2 - D_f} \frac{r_{\max}}{r_{\min}} \right)} \\ f = D_f r_{\min}^{D_f} r^{-(D_f + 1)} \\ L_{\tau} = (2r)^{1 - D_T} L^{D_T} \\ = (2r)^{1 - D_T} \left[ \frac{\pi D_f r_{\max}^2}{(2 - D_f) r_{\min}^{2 - D_f}} (r_{\max}^{2 - D_f} - r_{\min}^{2 - D_f}) \right]^{\frac{D_T}{2}} \end{array} \right. \quad (2)$$

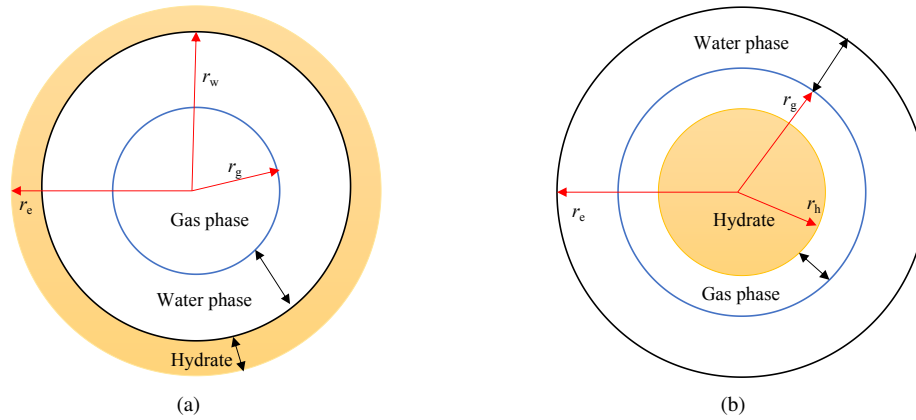
where  $D_{fn}$  is the pore fractal dimension of HBS under effective stress  $p_{eff} = 0$ ;  $\varphi$  is the porosity; and the subscript  $maxn$  means the maximum value of HBS under effective stress  $p_{eff} = 0$ .

Eq. (1) reveals that residual water saturation  $S_{wc}$  changes with the change in hydrate saturation  $S_h$ . Specifically, for a given parameter  $r_{c0}$ , since parameters  $r_{\max}$  and  $r_{\min}$  decrease with the increasing of  $S_h$ . Eq. (1) suggests that residual water saturation will increase with the increasing of hydrate as parameter  $S_h$  increases. Physically, the critical pore radius  $r_{c0}$  in Eq. (1) is the function of varies parameters (clay minerals, clay content, and fluid physical properties). Based on Eq. (1), the critical pore radius  $r_{c0}$  is

$$r_{c0} = \left[ (r_{\max}^{3-D_T-D_f} - r_{\min}^{3-D_T-D_f}) S_{wc} + r_{\min}^{3-D_T-D_f} \right]^{\frac{1}{3-D_T-D_f}} \quad (3a)$$

In this paper, we assume  $r_{c0}$  is independent on the effective stress, then, based on Eq. (3a), we have

$$r_{c0} = \left[ (r_{\max n}^{3-D_{Tn}-D_{fn}} - r_{\min n}^{3-D_{Tn}-D_{fn}}) S_{wc0} + r_{\min n}^{3-D_{Tn}-D_{fn}} \right]^{\frac{1}{3-D_{Tn}-D_{fn}}} \quad (3b)$$



**Fig. 3.** Schematic of two main hydrate-growth patterns in hydrate-bearing sediments: (a) wall coating hydrates; (b) pore filling hydrates.

where  $S_{wc0}$  is the initial residual water saturation;  $D_{Tn}$  is the tortuosity fractal dimension of HBS under effective stress  $p_{eff} = 0$ ; the subscript minn means the minimum value of HBS under effective stress  $p_{eff} = 0$ . More details about these parameters can be found in the former literature (Yu and Li, 2001; Yu and Cheng, 2002; Lei et al., 2017, 2019a, 2019b).

Singh et al. (2019) suggested that the amount of clays in HBS strongly affected the magnitude of bound-water saturation, thus affected the critical pore radius. Velocity equations for gas phase and water phase in a capillary with radius  $r_e$  will be :

$$\begin{cases} v_g = \frac{-\Delta p}{\mu_g L \tau} \frac{r^2}{4} + c_1 \ln r + c_2 \\ v_w = \frac{-\Delta p}{\mu_w L \tau} \frac{r^2}{4} + c_3 \ln r + c_4 \end{cases} \quad (4)$$

where  $v_g$  and  $v_w$  are gas phase velocity and water phase velocity, respectively;  $c_1, c_2, c_3,$  and  $c_4$  are integration constants; and  $\mu_g$  and  $\mu_w$  are gas phase viscosity and water phase viscosity, respectively.

Boundary conditions for WC hydrates are:

$$\frac{\partial v_g}{\partial r} = 0 \quad @r \rightarrow 0 \Rightarrow c_1 = 0 \quad (5a)$$

$$\begin{aligned} v_g = v_w @r = r_g &\Rightarrow \frac{-\Delta p}{\mu_g L \tau} \frac{r_g^2}{4} + c_1 \ln r_g + c_2 \\ &= \frac{-\Delta p}{\mu_w L \tau} \frac{r_g^2}{4} + c_3 \ln r_g + c_4 \end{aligned} \quad (5b)$$

$$\tau_g = \left( \mu_g \frac{\partial v_g}{\partial r} \right) = \tau_w = \left( \mu_w \frac{\partial v_w}{\partial r} \right) \quad (5c)$$

$$@r = r_g \Rightarrow \frac{-\Delta p r_g}{2L\tau} + \frac{\mu_g c_1}{r_g} = \frac{-\Delta p r_g}{2L\tau} + \frac{\mu_w c_3}{r_g}$$

$$v_w = 0 \quad @r = r_w \Rightarrow \frac{-\Delta p}{\mu_g L \tau} \frac{r_w^2}{4} + c_3 \ln r_w + c_4 = 0 \quad (5d)$$

and boundary conditions for PF hydrates are:

$$v_g = 0 \quad @r = r_h \Rightarrow \frac{-\Delta p}{\mu_g L \tau} \frac{r_h^2}{4} + c_1 \ln r_h + c_2 = 0 \quad (5e)$$

$$\begin{aligned} v_g = v_w \quad @r = r_g &\Rightarrow \frac{-\Delta p}{\mu_g L \tau} \frac{r_g^2}{4} + c_1 \ln r_g + c_2 \\ &= \frac{-\Delta p}{\mu_w L \tau} \frac{r_g^2}{4} + c_3 \ln r_g + c_4 \end{aligned} \quad (5f)$$

$$\tau_g = \left( \mu_g \frac{\partial v_g}{\partial r} \right) = \tau_w = \left( \mu_w \frac{\partial v_w}{\partial r} \right) \quad (5g)$$

$$@r = r_g \Rightarrow \frac{-\Delta p r_g}{2L\tau} + \frac{\mu_g c_1}{r_g} = \frac{-\Delta p r_g}{2L\tau} + \frac{\mu_w c_3}{r_g}$$

$$v_w = 0 \quad @r = r_e \Rightarrow \frac{-\Delta p}{\mu_g L \tau} \frac{r_e^2}{4} + c_3 \ln r_e + c_4 = 0 \quad (5h)$$

where  $r$  is the radius, and the subscripts  $h, g,$  and  $w$  denotes hydrates, gas and water, respectively.  $\mu_g$  and  $\mu_w$  can be determined as (Gupta et al., 2017):

$$\begin{cases} \mu_g = 10.4 \times 10^{-6} \left( \frac{273.15 + 162}{T + 162} \right) \left( \frac{T}{273.15} \right)^{1.5} \\ \mu_w = 0.001792 \exp \left[ -1.94 - 4.80 \left( \frac{T}{273.15} \right) \right. \\ \left. + 6.74 \left( \frac{T}{273.15} \right)^2 \right] \end{cases} \quad (6)$$

where  $T$  is temperature. Eq. (6) implies that  $\mu_g$  is much lower than  $\mu_w$ .

From Eqs. (4) and (5), the flow velocity equations for WC hydrates and PF hydrates are:

$$\begin{cases} v_g^{WC} = \frac{-\Delta p}{\mu_g L \tau} \frac{r^2 - r_g^2}{4} + \frac{-\Delta p}{\mu_w L \tau} \frac{r_g^2 - r_w^2}{4} & 0 < r \leq r_g \\ v_w^{WC} = \frac{-\Delta p}{\mu_w L \tau} \frac{r^2 - r_w^2}{4} & r_g < r \leq r_w \end{cases} \quad (7)$$

and

$$\left\{ \begin{aligned} v_g^{PF} &= \frac{-\Delta p}{\mu_g L \tau} \frac{r^2 - r_h^2}{4} + \frac{\Delta p}{4\mu_g L \tau} \frac{r_g^2 - r_h^2 + \alpha(r_e^2 - r_g^2)}{\ln(r_g/r_h) + \alpha \ln(r_e/r_g)} \ln \frac{r}{r_h} \\ & \quad r_h \leq r \leq r_g \\ v_w^{PF} &= \frac{-\Delta p}{\mu_w L \tau} \frac{r^2 - r_e^2}{4} + \frac{\alpha \Delta p}{4\mu_g L \tau} \frac{r_g^2 - r_h^2 + \alpha(r_e^2 - r_g^2)}{\ln(r_g/r_h) + \alpha \ln(r_e/r_g)} \ln \frac{r}{r_e} \\ & \quad r_g \leq r \leq r_e \end{aligned} \right. \quad (8)$$

where  $\alpha = \mu_g/\mu_w$  is the viscosity ratio. Integrating Eqs. (7) and (8), the gas-water volumetric flow rates in a single capillary for WC hydrates and PF hydrates are:

$$\left\{ \begin{aligned} q_g^{WC} &= \int_0^{r_g} 2\pi r v_g dr = \frac{\Delta p \pi r_g^4}{8\mu_g L \tau} \left[ 1 - 2\alpha \left( 1 - \frac{r_w^2}{r_g^2} \right) \right] \\ q_w^{WC} &= \int_{r_g}^{r_w} 2\pi r v_w dr = \frac{\Delta p \pi}{8\mu_w L \tau} (r_g^2 - r_w^2)^2 \end{aligned} \right. \quad (9)$$

and

$$\left\{ \begin{aligned} q_g^{PF} &= \int_{r_h}^{r_g} 2\pi r v_g^{PF} dr \\ &= \frac{\Delta p \pi}{8\mu_g L \tau} \left\{ -(r_g^2 - r_h^2)^2 + \frac{r_g^2 - r_h^2 + \alpha(r_e^2 - r_g^2)}{\ln(r_g/r_h) + \alpha \ln(r_e/r_g)} \right. \\ & \quad \left. \times \left[ 2r_g^2 \ln \left( \frac{r_g}{r_h} \right) - (r_g^2 - r_h^2) \right] \right\} \\ q_w^{PF} &= \int_{r_g}^{r_e} 2\pi r v_w^{PF} dr \\ &= \frac{\Delta p \pi}{8\mu_w L \tau} \left\{ (r_e^2 - r_g^2)^2 + \frac{r_g^2 - r_h^2 + \alpha(r_e^2 - r_g^2)}{\ln(r_g/r_h) + \alpha \ln(r_e/r_g)} \right. \\ & \quad \left. \times \left[ 2r_g^2 \ln \left( \frac{r_g}{r_h} \right) - (r_e^2 - r_g^2) \right] \right\} \end{aligned} \right. \quad (10)$$

Based on Eqs. (2), (9) and (10), the total gas-water volumetric flow rates for WC hydrates and PF hydrates in HBS are (Yu et al., 2003; Lei et al., 2019b):

$$\left\{ \begin{aligned} Q_g^{WC} &= \left( \frac{r_{\max}}{r_{\min}} \right)^{D_f} \int_{r_{c0}}^{r_{\max}} f(r_e) q_g^{WC} dr_e \\ &= \frac{\pi \Delta p D_f r_{\max}^{D_f}}{\mu_g 2^{4-D_f} L D_T} \int_{r_{c0}}^{r_{\max}} r_g^4 \left[ 1 - 2\alpha \left( 1 - \frac{r_w^2}{r_g^2} \right) \right] r_e^{D_T - D_f - 2} dr_e \\ Q_w^{WC} &= \left( \frac{r_{\max}}{r_{\min}} \right)^{D_f} \int_{r_{c0}}^{r_{\max}} f(r_e) q_w^{WC} dr_e \\ &= \frac{\pi \Delta p D_f r_{\max}^{D_f}}{\mu_w 2^{4-D_f} L D_T} \int_{r_{c0}}^{r_{\max}} (r_g^2 - r_w^2)^2 r_e^{D_T - D_f - 2} dr_e \end{aligned} \right. \quad (11)$$

and

$$\left\{ \begin{aligned} Q_g^{PF} &= N \int_{r_{c0}}^{r_{\max}} q_g^{PF} f(r_e) dr_e \\ &= \frac{\pi \Delta p D_f r_{\max}^{D_f}}{\mu_g 2^{4-D_f} L D_T} \int_{r_{c0}}^{r_{\max}} r_e^{D_T - D_f - 2} \\ & \quad \left\{ -(r_g^2 - r_h^2)^2 + \frac{r_g^2 - r_h^2 + \alpha(r_e^2 - r_g^2)}{\ln(r_g/r_h) + \alpha \ln(r_e/r_g)} \right. \\ & \quad \left. \left[ 2r_g^2 \ln \left( \frac{r_g}{r_h} \right) - (r_g^2 - r_h^2) \right] \right\} dr_e \\ Q_w^{PF} &= N \int_{r_{c0}}^{r_{\max}} q_w^{PF} f(r_e) dr_e \\ &= \frac{\pi \Delta p D_f r_{\max}^{D_f}}{\mu_w 2^{4-D_f} L D_T} \int_{r_{c0}}^{r_{\max}} r_e^{D_T - D_f - 2} \\ & \quad \left\{ (r_w^2 - r_g^2)^2 + \frac{r_g^2 - r_h^2 + \alpha(r_e^2 - r_g^2)}{\ln(r_g/r_h) + \alpha \ln(r_e/r_g)} \right. \\ & \quad \left. \left( 2r_g^2 \ln \left( \frac{r_g}{r_h} \right) - (r_w^2 - r_g^2) \right) \right\} dr_e \end{aligned} \right. \quad (12)$$

According to the distribution of three phases in a capillary with radius  $r$  larger than  $r_{c0}$  (Fig. 3), the saturations of hydrates, gas, and water in a capillary for WC hydrates and PF hydrates can be expressed as:

$$\left\{ \begin{aligned} S_{he}^{WC} &= 1 - \frac{r_w^2}{r_e^2}, \quad S_{ge}^{WC} = \frac{r_g^2}{r_e^2}, \quad S_{we}^{WC} = \frac{r_w^2 - r_g^2}{r_e^2} \\ S_{he}^{PF} &= \frac{r_h^2}{r_e^2}, \quad S_{ge}^{PF} = \frac{r_g^2 - r_h^2}{r_e^2}, \quad S_{we}^{PF} = 1 - \frac{r_g^2}{r_e^2} \end{aligned} \right. \quad (13)$$

Using Eq. (13), Eqs. (11) and (12) can be rearranged as:

$$\left\{ \begin{aligned} Q_g^{WC} &= \frac{\pi \Delta p D_f r_{\max}^{D_f} [(S_{ge}^{WC})^2 + 2\alpha S_{ge}^{WC} S_{we}^{WC}]}{\mu_g 2^{4-D_f} L D_T} \\ & \quad \times \frac{r_{\max}^{D_T - D_f + 3} - r_{c0}^{D_T - D_f + 3}}{D_T - D_f + 3} \\ Q_w^{WC} &= \frac{\pi \Delta p D_f r_{\max}^{D_f} (S_{we}^{WC})^2 r_{\max}^{D_T - D_f + 3} - r_{c0}^{D_T - D_f + 3}}{\mu_w 2^{4-D_f} L D_T} \frac{r_{\max}^{D_T - D_f + 3} - r_{c0}^{D_T - D_f + 3}}{D_T - D_f + 3} \end{aligned} \right. \quad (14)$$

and

$$\left\{ \begin{aligned} Q_g^{PF} &= \frac{\pi \Delta p D_f r_{\max}^{D_f}}{\mu_g 2^{4-D_f} L D_T} \\ & \quad \times \int_{r_{c0}}^{r_{\max}} r^{D_T - D_f + 2} \{ -(S_{ge}^{PF})^2 + MM[MN_0 - S_{ge}^{PF}] \} dr \\ Q_w^{PF} &= \frac{\pi \Delta p D_f r_{\max}^{D_f}}{\mu_w 2^{4-D_f} L D_T} \\ & \quad \times \int_{r_{c0}}^{r_{\max}} r^{D_T - D_f + 2} \{ (S_{we}^{PF})^2 + MM[MN_1 - S_{we}^{PF}] \} dr \end{aligned} \right. \quad (15)$$

where

$$\begin{cases} MM = \frac{S_{ge}^{PF} + \alpha S_{we}^{PF}}{\ln \left[ \sqrt{\frac{S_{he}^{PF} + S_{ge}^{PF}}{S_{he}^{PF}}} \right] + \alpha \ln \left[ \sqrt{\frac{1}{S_{he}^{PF} + S_{ge}^{PF}}} \right]} \\ MN_0 = 2(S_{he}^{PF} + S_{ge}^{PF}) \ln \left( \sqrt{\frac{S_{he}^{PF} + S_{ge}^{PF}}{S_{he}^{PF}}} \right) \\ MN_1 = 2(S_{he}^{PF} + S_{ge}^{PF}) \ln \left( \sqrt{\frac{1}{S_{he}^{PF} + S_{ge}^{PF}}} \right) \end{cases} \quad (16)$$

Based on Eqs. (14) to (16) and absolute permeability stated in the appendix A, the gas-water relative permeability for WC hydrates and PF hydrates in porous media is:

$$\begin{cases} K_{rg}^{WC} = (S_{ge}^{WC})^3 + 2\alpha(S_{ge}^{WC})^2 S_{we}^{WC} \\ K_{rw}^{WC} = (S_{we}^{WC})^3 \end{cases} \quad (17)$$

and

$$\begin{cases} K_{rg}^{PF} = S_{ge}^{PF} \left\{ -(S_{ge}^{PF})^2 + MM[MN_0 - S_{ge}^{PF}] \right\} \\ K_{rw}^{PF} = S_{we}^{PF} \left\{ (S_{we}^{PF})^2 + MM[MN_1 - S_{we}^{PF}] \right\} \end{cases} \quad (18)$$

The saturations of hydrates, gas, and water in porous media are:

$$\begin{cases} S_h = S_{he}^{WC} (1 - S_{wc}) & S_g = S_{ge}^{WC} (1 - S_{wc}) \\ S_w = S_{we}^{WC} (1 - S_{wc}) + S_{wc} \\ S_h = S_{he}^{PF} (1 - S_{wc}) & S_g = S_{ge}^{PF} (1 - S_{wc}) \\ S_w = S_{we}^{PF} (1 - S_{wc}) + S_{wc} \end{cases} \quad (19)$$

Inserting Eq. (19) into Eqs. (16), (17) and (18), we have:

$$\begin{cases} K_{rg}^{WC} = \left( \frac{S_g}{1 - S_{wc}} \right)^3 + 2\alpha \left( \frac{S_g}{1 - S_{wc}} \right)^2 \frac{S_w - S_{wc}}{1 - S_{wc}} \\ K_{rw}^{WC} = \left( \frac{S_w - S_{wc}}{1 - S_{wc}} \right)^3 \end{cases} \quad (20)$$

and

$$\begin{cases} K_{rg}^{PF} = \frac{S_g}{1 - S_{wc}} \left\{ - \left( \frac{S_g}{1 - S_{wc}} \right)^2 + MM \left( MN_0 - \frac{S_g}{1 - S_{wc}} \right) \right\} \\ K_{rw}^{PF} = \frac{S_w - S_{wc}}{1 - S_{wc}} \left\{ \left( \frac{S_w - S_{wc}}{1 - S_{wc}} \right)^2 + MM \left( MN_1 - \frac{S_w - S_{wc}}{1 - S_{wc}} \right) \right\} \end{cases} \quad (21)$$

where

$$\begin{cases} MM = \frac{1}{1 - S_{wc}} \frac{S_g + \alpha(S_w - S_{wc})}{\ln \left( \sqrt{1 + \frac{S_g}{S_h}} \right) - \alpha \ln \left( \sqrt{\frac{1 - S_w}{1 - S_{wc}}} \right)} \\ MN_0 = 2 \frac{1 - S_w}{1 - S_{wc}} \ln \left( \sqrt{1 + \frac{S_g}{S_h}} \right) \\ MN_1 = -2 \frac{1 - S_w}{1 - S_{wc}} \ln \left( \sqrt{\frac{1 - S_w}{1 - S_{wc}}} \right) \end{cases} \quad (22)$$

Eqs. (20) through (22) show that the  $K_{rw}$  (or  $K_{rg}$ ) is a function of residual water saturation, water saturation (or gas saturation), hydrate saturation, and viscosity ratio related to temperature. Then, by using a relative permeability statistical average method (Sing et al., 2019a, 2019b), the effective relative permeability for describing different hydrate-growth patterns can be expressed as:

$$\begin{cases} K_{rg} = (K_{rg}^{PF})^\lambda (K_{rg}^{WC})^{1-\lambda} \\ K_{rw} = (K_{rw}^{PF})^\lambda (K_{rw}^{WC})^{1-\lambda} \end{cases} \quad (23)$$

where  $\lambda$  is the proportion of PF in all growth patterns of hydrates, which can be determined by experimental tests. Physically, parameter  $\lambda$  varies in the range of 0 (i.e., WC hydrate) to 1 (i.e., PF hydrate). Moreover, parameter  $\lambda$  increases with the increasing of hydrate saturation (i.e., WC hydrates will change into PF hydrates as hydrate saturation increases).

### 3. Validation

In this section, the derived models are validated by comparing the calculated results to different experimental data sets. Firstly, a former model by Lei et al. (2015) are selected to validate our derived relative permeability model with hydrate saturation  $S_h = 0$ . Additionally, experimental results of  $K_{rw}$  in HBS are also utilized to validate the relative permeability model in HBS.

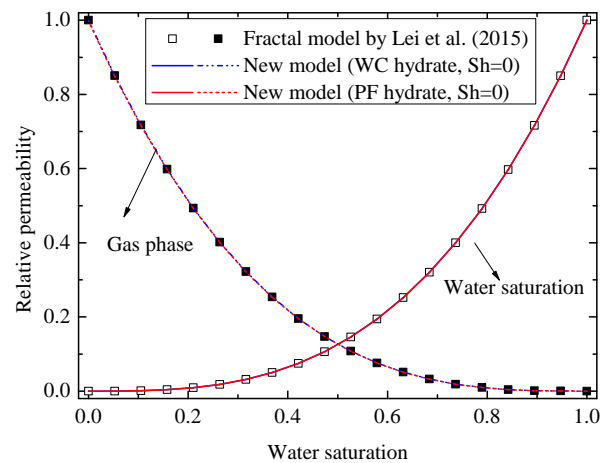


Fig. 4. Gas-water relative permeability in dry porous media without gas hydrates.

### 3.1 Gas-water relative permeability in dry porous media

Fig. 4 presents the comparison of the results from the fractal model by Lei et al. (2015) and the predictions from our derived model. When the immobile liquid film thickness  $\delta$  in the fractal model is set to be 0, the fractal model can be used to calculate  $K_{rw}$  and  $K_{rg}$  in dry porous media without gas hydrates. In our derived model, the initial residual water saturation  $S_{wc0}$  is set to 0, the hydrate parameter  $S_h$  is set to 0, and  $p_{eff}$  is set to 0. The results (Fig. 4) indicate that the developed model is validated.

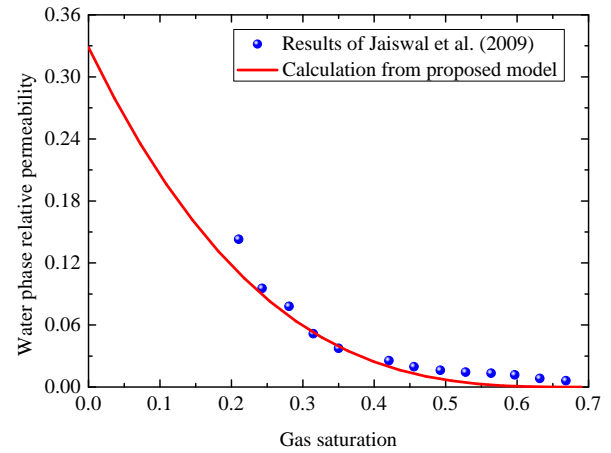
### 3.2 Water phase relative permeability in HBS

To further verify the derived model, we used experimental data from Jaiswal et al. (2009), who performed unsteady state gas-water relative permeability measurements on a synthetic sandstone sample with gas hydrate saturation of 0.31 in the Anadarko field. In the test, 2% brine was used to saturate the consolidated synthetic sandstone sample under a flow rate of 0.04 ml/min. Then, high pressure cold methane of temperature 4.5 °C and pressure 950 ~ 1250 psig (6.55 ~ 8.62) MPa was injected into the sample with the optimal rate of 1.5 ml/min to form gas hydrates. After the formation of hydrates, porosity and permeability of porous media were 7.85% and  $7.04 \times 10^{-5} \mu\text{m}^2$ , respectively. Then, gas-water displacement tests were performed with the unsteady state method. In our proposed model, the parameter porosity  $\phi$  is set to 7.85%, the initial residual water saturation is set to 0, and the effective stress is set to 0. In addition, the initial maximum radius  $r_{\text{maxn}} = 0.47 \mu\text{m}$ , the initial minimum radius  $r_{\text{minn}} = 4.7 \times 10^{-3} \mu\text{m}$ , and hydrate saturation  $S_h = 31\%$  are assigned to our model, which could ensure that the parameter  $K$  is  $7.04 \times 10^{-5} \mu\text{m}^2$ . Fig. 5 compares the measured  $K_{rw}$  to the predicted data. As shown in Fig. 5, the predicted results provide a good match over the entire range of data.

## 4. Discussion

After validation by exhaustive experimental data, this derived model was used for sensitivity analysis of relevant physical parameters (e.g., hydrate saturation, effective stress, initial residual water saturation, etc.)  $K_{rw}$  and  $K_{rg}$  of HBS under stress dependence.

Fig. 6 shows the relative permeability versus gas saturation with different hydrate saturation. One can also see that gas-water relative permeability in PF hydrates is generally lower than that in WC. One can also see that, under the same saturation,  $K_{rg}$  and  $K_{rw}$  in PF hydrates are generally lower than those in WC hydrates. Fig. 6(a) also reveals that, for WC hydrates (i.e.,  $\lambda = 0$ ), with a given gas saturation, a larger  $K_{rw}$  corresponds to a lower hydrate saturation, whereas the effect of hydrate  $S_h$  on  $K_{rg}$  is not significant. In contrast, Fig. 6(b) shows that, for PF hydrates (i.e.,  $\lambda = 1$ ), with a given gas saturation, both water phase relative permeability and gas phase relative  $K_{rw}$  and  $K_{rg}$  decline as hydrate saturation increases. In general, with the increasing of  $S_h$ , the pore sizes will become narrower,



**Fig. 5.** Relative permeability of water in porous media in the presence of hydrates. Effective stress is set to 0, parameter  $\lambda$  is set to 0 (i.e., WC hydrates), and parameter  $D_{fn}$  is set to 1.45.

and the wettability of water phase increases. Physically, with increasing wetting by the water, the intersection of the non-wetting and water relative permeabilities shifts to smaller nonwetting phase saturation (or the larger water saturation), and the maximum  $K_{rw}$  decreases. Similar results are shown in Fig. 6. In addition, Fig. 6(b) shows that  $K_{rw}$  firstly increases and then decreases as gas saturation increases. This can be interpreted as that, during gas-water flow in HBS, the water phase (i.e., fluid with relative higher viscosity) flows past the gas phase (i.e., fluid with relative lower viscosity). With lower gas saturation, the flow of the water phase, to some extent, can be regarded as a sliding motion in which low-viscosity fluid (gas phase) provides lubrication (Odeh, 1959; Ehrlich, 1993; Shad and Gates, 2010) (see Fig. 3(b)). For WC hydrates, however,  $K_{rw}$  monotonically decreases with the increasing of gas saturation, because the gas phase is no longer between the water phase and the hydrates (see Fig. 3(a)).

Since the parameter  $S_{wc0}$ , to some extent, represents the relative strength of wettability (Singh, 2019a), the derived model can be applied to investigate the effect of wettability on relative permeability. Fig. 7 shows relative permeability curves with different initial residual water saturations for hydrate saturations equal to 0.2 and 0.4. It is clear that a larger initial residual water saturation corresponds to a higher  $K_{rg}$  and a lower  $K_{rw}$ . Results presented in Fig. 7 also suggest that  $K_{rg}$  increases as wettability increases, while  $K_{rw}$  decreases as wettability increases. Similar observations were also reported in (Singh, 2019a). Moreover, it can also be seen from Fig. 7 that both  $K_{rg}$  and  $K_{rw}$  decline as  $S_h$  increases. Similar results are shown in Fig. 6.

Fig. 8 presents relative permeability curves with different effective stress (for  $0 \leq p_{eff} \leq 15$  MPa). In our proposed model, residual water saturation is set to 0.1, rock elastic modulus is set to 8 GPa, and Poisson's ratio is set to 0.25. Fig. 8 shows that, under the given effective stress,  $K_{rg}$  increases with increasing effective stress, while  $K_{rw}$  decreases with the increase of  $p_{eff}$ . This finding is consistent with observations in other theoretical and experimental works (Lei et al., 2017).

Fig. 9 presents relative permeability curves with different

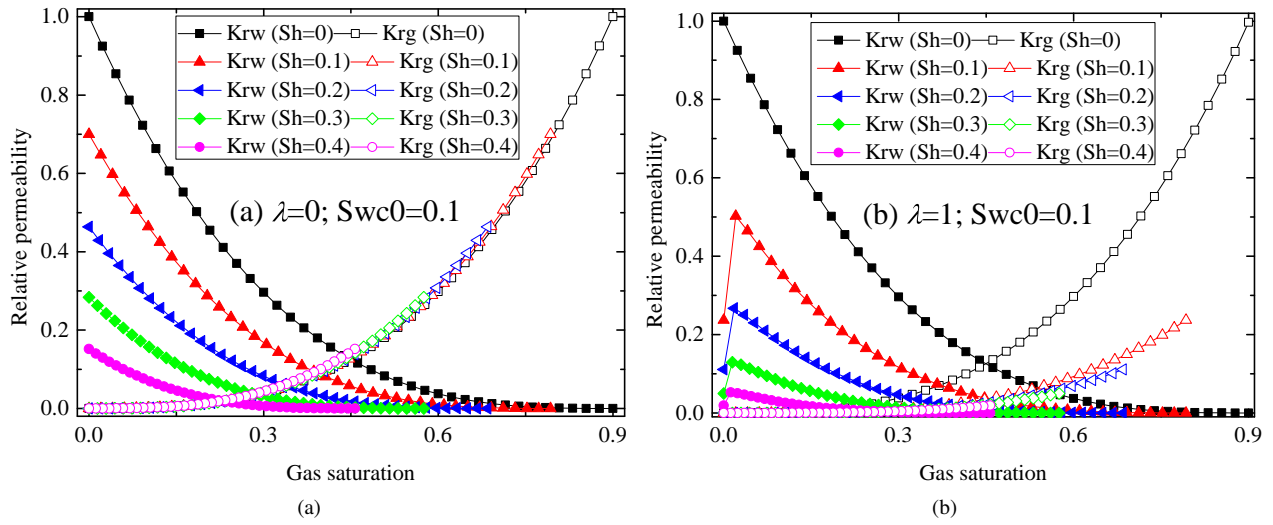


Fig. 6. Relative permeability versus gas saturation in HBS with different hydrate saturations. Effective stress is set to 0, and initial residual water saturation is set to 0.1.

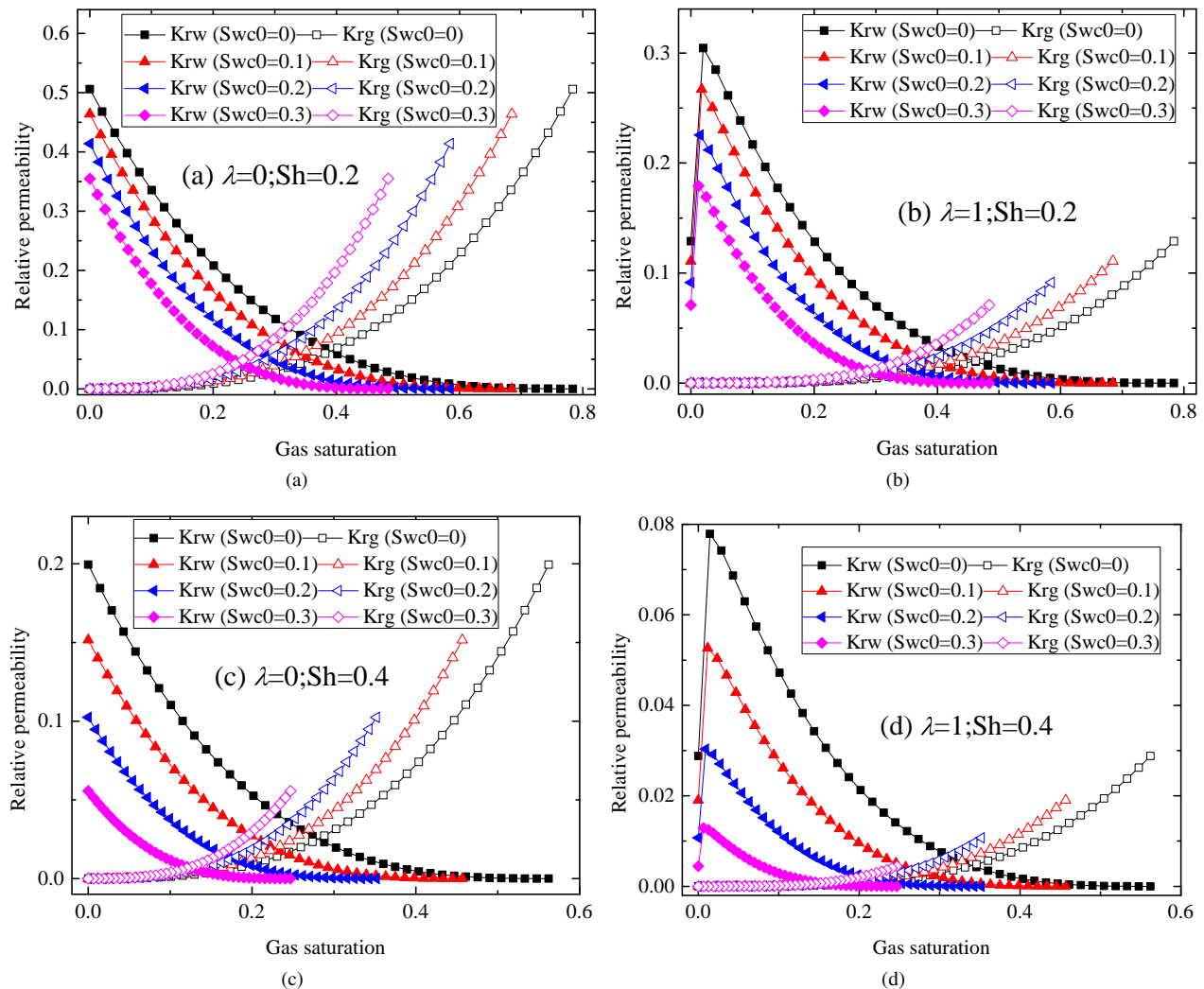
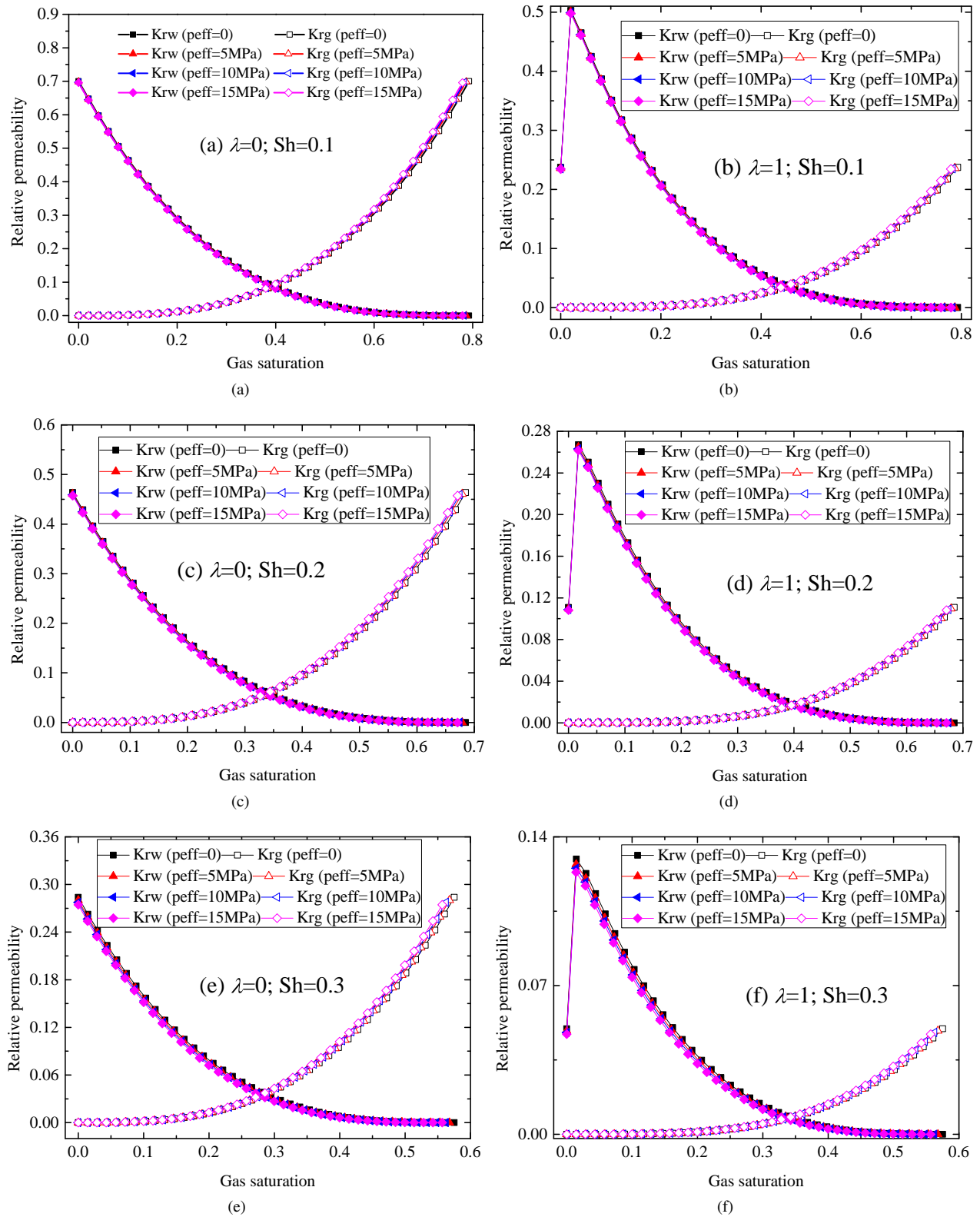
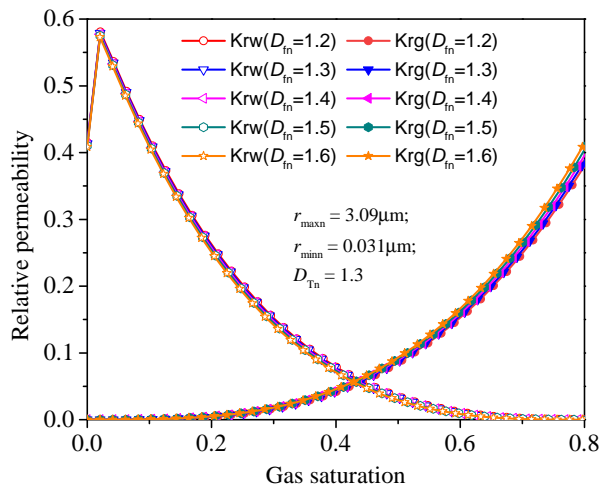


Fig. 7. Relative permeability versus gas saturation in porous media with different initial residual water saturation. Effective stress is set to 0.





**Fig. 8.** Relative permeability versus gas saturation in porous media with different effective stress. Initial residual water saturation is set to 0.1.



**Fig. 9.** Relative permeability in HBS with different  $D_{fn}$ . Hydrate saturation is set to 0.1, initial residual water saturation is set to 0.1, parameter  $\lambda$  is set to 0.5.

parameter  $D_{fn}$  (for  $1.2 \leq D_{fn} \leq 1.6$ ). In the modeling, relevant parameters are assigned as follows:  $r_{\max n} = 3.09 \mu\text{m}$ ;  $r_{\min n} = 0.031 \mu\text{m}$ ;  $D_{Tn} = 1.3$ ; effective stress  $p_{eff} = 10 \text{ MPa}$ ; rock elastic modulus  $E = 14.4 \text{ GPa}$ , and Poisson's ratio  $\nu = 0.25$ . As seen from Fig. 9, for a specific fluid saturation,  $K_{rw}$  decreases as the  $D_{fn}$  increases, while  $K_{rg}$  increases as the  $D_{fn}$  increases. The reason is that, a larger  $D_{fn}$  replies a more complex pore structure and a larger percent of small pores, leading to larger water phase flow resistance. Similar findings can be seen in other theoretical works (Xu et al., 2013; Lei et al., 2015).

## 5. Advantages and limitations of the derived model

The proposed relative permeability predictive model lays theoretical foundations for characterizing gas-water flow in deformable HBS. In addition, by combing inverse modeling, it can help to determine the pore-scale parameters with higher accuracy and the rock lithology. However, it is limited to capillary bundle model. Moreover, the derived model only accounts for PF hydrates and wall WC hydrates, and ignores other types (i.e., GC hydrates, lenses, and veins).

Results from this study suggest that, the effect of parameter  $\lambda$  (the proportion of pore filling in all growth patterns of hydrates) on the relative permeability prediction is significant. Physically, with the increasing of hydrate saturation, WC hydrates will change into PF hydrates. Thus, parameter  $\lambda$  increases as hydrate saturation increases. In general, experiment test is a good way to get the actual results. However, parameter  $\lambda$  in this work is predicted using the inverse modeling, and the quantitative relation between parameter  $\lambda$  and hydrate saturation is not determined. As a result, in this study,  $\lambda$  may cause uncertainty in the prediction, even though our predicted relative permeability is agreement with the test data. Consequently, further researches are required to reduce the uncertainty and make the model more accurate.

Moreover, in general, the residual water saturation changed dynamically with the change in hydrate saturation. However, in

this work, However, the quantitative relation between residual water saturation and hydrate saturation is not determined in previous literatures. Mathematically, our derived model Eq. (1) reveals that, for a given parameter  $r_{c0}$ , residual water saturation will increase with the increasing of hydrate saturation. Physically, parameter  $r_{c0}$  is related to hydrate saturation. However, the quantitative relation between parameter  $r_{c0}$  and hydrate saturation is not determined in this model. Consequently, further researches are needed to make the derived model more accurate.

## 6. Conclusion

In this paper, an analytical model is developed to investigate the stress-dependent water-gas relative permeability of hydrate-bearing sediments, which is validated by various experimental data and is shown to perform well. The existing relative permeability model without hydrate saturation is a special case of the derived model.

Gas-water relative permeability behavior in PF hydrates is significantly different from that in wall-coating WC hydrates. In addition, both gas phase and water phase relative permeabilities decline as hydrate saturation increases. As wettability increases,  $K_{rw}$  (relative permeability to wetting phase) decreases, while  $K_{rg}$  (relative permeability to non-wetting phase) increases. These findings suggest that, due to many factors in this complex system, we should be cautious in analyzing fluid flow in HBS.

In addition to providing theoretical foundations for quantifying relative permeability in HBS, another advantage of this derived stress-dependent permeability model is that, with experimental permeability data, it can be utilized to estimate pore-scale parameters and rock lithology using inverse modeling. Furthermore, the derived model can be applied as an alternative method for residual water saturation of deformable HBS determination, as nuclear magnetic resonance and special core analysis could be expensive and challenging. This work would provide a guidance to gas hydrates development.

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## Conflict of interest

The authors declare no competing interest.

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## Appendix A: Fractal permeability model of HBS under stress conditions

Based on fractal theory, absolute permeability  $K$  of porous media is (Xu et al., 2013; Lei et al., 2017)

$$\left\{ K = \frac{D_f r_{\max}^{D_f - D_T - 1} \pi}{2^{4 - D_T} \left[ \frac{\pi D_f}{2 - D_f} \left( \frac{r_{\max}^{2 - D_f}}{r_{\min}^{2 - D_f}} - 1 \right) \right]^{\frac{D_T + 1}{2}}} \frac{r_{\max}^{3 + D_T - D_f} - r_{c0}^{3 + D_T - D_f}}{3 + D_T - D_f} \right. \quad (\text{A-1})$$

where the maximum (or minimum) stress dependent pore radius  $r_{\max}$  (or  $r_{\min}$ ) can be written as (Lei et al., 2019a, 2019b)

$$\left\{ \begin{array}{l} r_{\max}(P_{eff}) = r_{\maxn} \left\{ 1 - 4 \left[ \frac{3\pi(1 - \nu^2)P_{eff}}{4E} \right]^{\beta} \right\} \\ r_{\min}(P_{eff}) = r_{\minn} \left\{ 1 - 4 \left[ \frac{3\pi(1 - \nu^2)P_{eff}}{4E} \right]^{\beta} \right\} \end{array} \right. \quad (\text{A-2})$$

where  $E$  denotes the Young's modulus,  $\nu$  is the Poisson's ratio, and  $\beta$  denotes the parameter representing the rough pore surface.