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Original article

Well pattern optimization based on StoSAG algorithm

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development plan

Abstract:
The well pattern optimization in the oilfield is challenging and intricate work due to the heterogeneity of the permeability and viscosity. Traditionally, the well pattern optimization is conducted by comparing the results of several plans manually designed by the reservoir engineer, which is difficult to obtain the optimal well pattern. To address these challenges, a framework that integrates a reservoir simulator into the StoSAG algorithm is proposed. The well pattern operators proposed by Onwunalu and Durlofsy are applied to obtain the variations of the well pattern and used as the optimization variables. During the framework, the optimization variables are continuously adjusted by the StoSAG algorithm in order to obtain the optimal one which maximize the objective function value. The framework is applied to a synthetic reservoir. The results show that the StoSAG algorithm can be successfully applied in the well pattern optimization and remarkably improve the development effect. This method can be widely used in new oilfield development plan and offer reference for well pattern deployment.

1. Introduction

The deployment of the well pattern is one of the most important problems in the oilfield development. The optimal well pattern is necessary for high recovery efficiency and satisfactory economics while the poor well pattern will lead to the low oil production and economic benefits (Feng et al., 2012).

Most of the oilfields in China are clastic rock deposition with strong heterogeneity. The reservoir heterogeneity has an important influence on the productivity and recovery efficiency (Wang et al., 2017). Therefore, the reservoir heterogeneity should be taken into the consideration to obtain the optimal well pattern. Many literatures are concerned with this specific topic. Basically, methods applied in the well pattern optimization can be divided into two categories: the conventional well pattern optimization method and the well pattern optimization method based on optimization theory.

The conventional well pattern optimization method mainly includes two types: the method based on the experience of petroleum engineer and the method based on the reservoir theory.

For the first method, well pattern optimization is conducted by comparing several well pattern plans manually designed by petroleum engineer (Zhou et al., 2002). An et al. (2013) took the BZ oilfield as an example and developed several well pattern plans according to the distribution of the oil-bearing sand and the permeability. The optimal well pattern was obtained by comparing the results of the reservoir numerical simulation corresponding to the plan. Xu et al. (2014) studied the optimization of fracturing parameters in the well pattern with horizontal and vertical wells combined. Similarly, several plans were designed to obtain the optimal one.

As for the second method, many analytic expressions were developed according to the reservoir engineering theory. Zhou et al. (2008) derived the design formula of well space for reservoir with permeability heterogeneity based on porous flow theory. The validity of the formula is verified by the water flooding experiments.

In the majority of the research mentioned above, the method mainly depends on the experience of petroleum engineers and focused on the parametric analysis, so the application of this method is limited in a specific geology case.

With the development and application of the computer, the well pattern optimization is transformed into the optimization problem based on the optimization theory. With the aid of the reservoir numerical simulation, the well pattern optimization

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is to adjust the location of the producer and injector by the optimization algorithm to maximize the objective function value. The objective function value used in the well pattern optimization is the NPV (net present value) or the cumulative oil production (Brouwer and Jansen, 2004; Zhao et al., 2011; Do et al., 2012; Yao et al., 2012; Zhao et al., 2013; Awotunde, 2014; Oliveira and Reynolds, 2014; Fonseca, 2015). Yao et al. (2012) maximized the NPV to solve the constrained reservoir production optimization. Awotunde (2014) tried to obtain the optimal locations of wells and the optimal controls by maximizing the NPV.

The optimization algorithm can be divided into the gradient algorithm (Sarma and Chen, 2008; Essen et al., 2009, 2011; Zhang et al., 2010; Montleau et al., 2014) and gradient free algorithm (Spall, 1992, 1998, 2000; Beckner and Song, 1995; Guyaguler, 2002; Bangerth et al., 2006; Chen et al., 2008, 2010; Emerick et al., 2009; Wang, 2009; Onwunalu and Durlofsky, 2010; Feng et al., 2012; Li et al., 2013; Isebor et al., 2014; Zhang et al., 2015). Beckner and Song (1995) applied the simulated annealing algorithm (SA) to maximize the NPV and Emerick et al. (2009) tried the genetic algorithm (GA) to solve the well placement optimization with nonlinear constraints. Onwunalu and Durlofsky (2010), Feng et al. (2012) applied the particle swarm optimization algorithm (PSO) to the well pattern optimization. Spall (1992) developed the simultaneous perturbation stochastic approximation algorithm (SPSA). The gradient is obtained by disturbing the control vector and only the computation of objective function value is involved. Although the gradient here is stochastic, it can be guaranteed that the search direction is always uphill and the expectation is the true gradient for the maximization problem. Chen and Oliver (2008, 2010) proposed the EnOpt algorithm which proved to be applied successfully to solve reservoir production optimization. The EnOpt can not only be used to optimize the production of a single reservoir model, but also can be used for robust production optimization based on multiple models. Many control vectors obeying the Gauss distribution are firstly obtained based on the current optimal control vector and then the covariance of the control vectors and the objective function value corresponding to the control vectors are computed to determine the search direction.

The StoSAG algorithm is proposed by Fonseca et al. (2016), which has been widely applied to solve the optimization problem. In this paper, the StoSAG is applied to the well patter optimization. Firstly, the well pattern establishment and the well pattern optimization model are given, and then the StoSAG is applied to a synthetic reservoir example in order to check the validity of the method.

2. Well pattern establishment

The general well patterns in oilfield include five-spot pattern, seven-spot pattern and nine-spot pattern. Onwunalu and Durlofsky (2010) have put forward the solution of transforming the well pattern question into the optimization question. Firstly select a well pattern unit, then do a series of transforming operation to get the new well pattern, lastly expand the new well pattern on the whole reservoir.

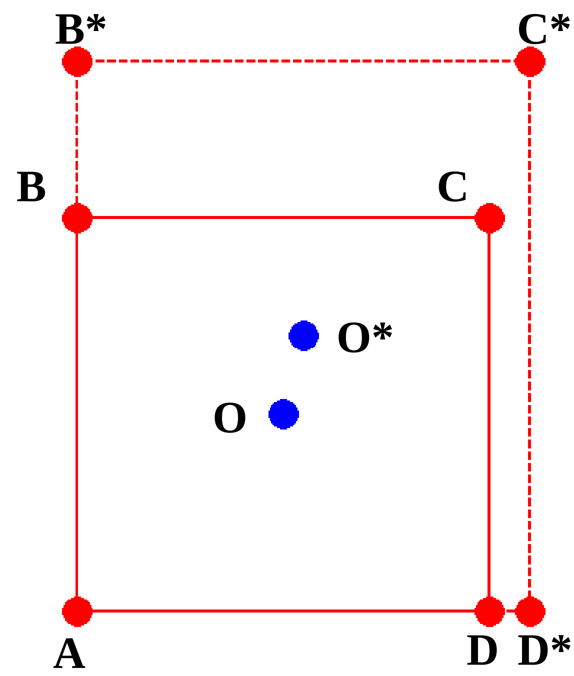


Fig. 1. Sketch of scaling transformation.

2.1 Well pattern operators

The well pattern operators raised by Onwunalu include the scaling, shift, shear and rotation. The five-spot well pattern is taken as example to illustrate the transformation process.

(1) Scaling

Scaling operator increases or decreases the size of a well pattern, thus the well pattern can change, as shown in Fig. 1.

(2) Shift

A well pattern unit can be moved by the shift operator without changing the original geometric shape, as shown in Fig. 2, while the well pattern unit moves horizontally and vertically by Δx and Δy, respectively.

(3) Shear

Through the shear operation on well pattern, the lateral deformation has been accomplished. As shown in Fig. 3, the well pattern changes the shape by shearing a specific angle γ.

(4) Rotation

Set one peak of well pattern as the center point and rotate a certain angle clockwise, the well pattern rotation will be accomplished. As shown in Fig. 4, set point A as the center point and rotate θ degrees, then the well pattern has changed.

After the scaling, shift, shear and rotation transformation, the new well pattern is transformed from the initial well pattern ABCD to the final well pattern A*B*C*D* shown in Fig. 5.

The four operators are set as the well pattern vector $u = \{asf,\ bsf,\ \Delta x,\ \Delta y,\ \gamma,\ \theta\}$, where asf, bsf are the scaling factor, Δx, Δy are the length of shift, γ is the shear angle and θ is the rotating angle. The component of the well pattern vector has the certain limits:

$$asf \in \left[0, \max\left(\frac{RS - xRef}{a_0}, \frac{LS - xRef}{a_0}\right)\right] \tag{1}$$

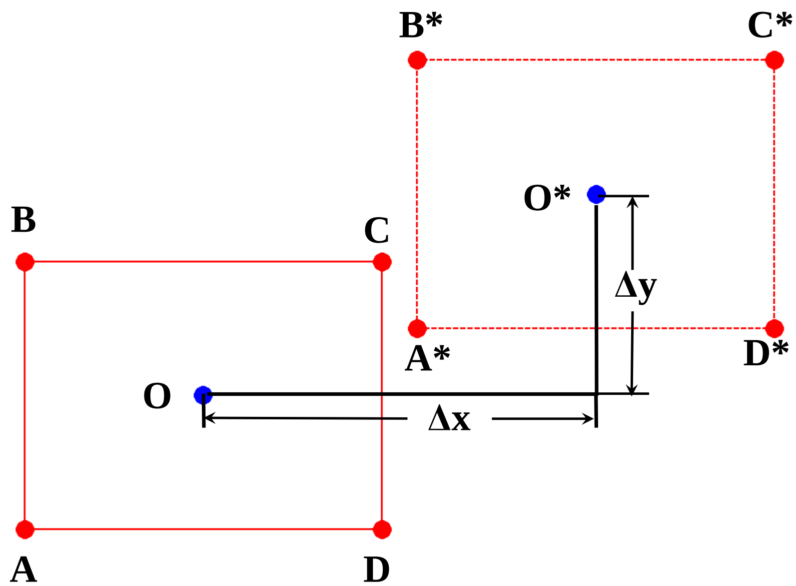


Fig. 2. Sketch of shift transformation.

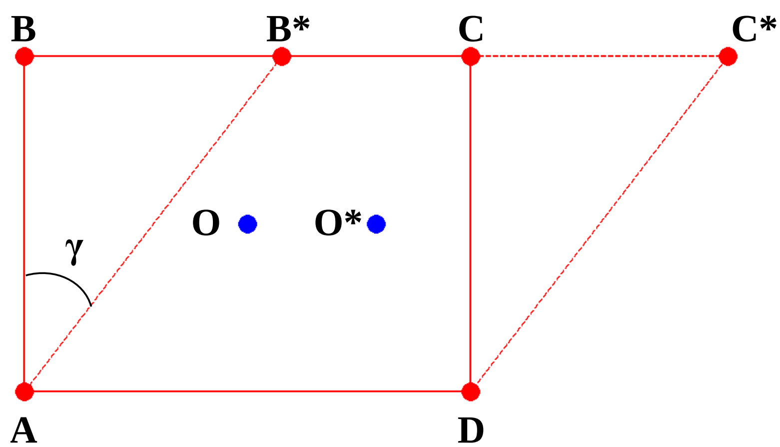


Fig. 3. Sketch of shear transformation.

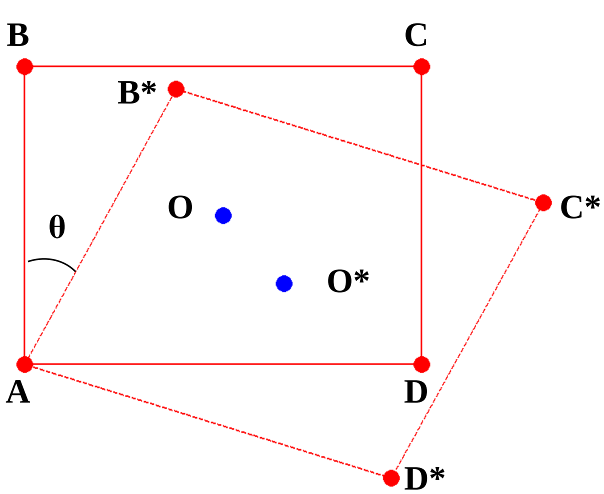


Fig. 4. Sketch of rotation transformation.

$$bsf \in \left[0, \max\left(\frac{US - yRef}{b_0}, \frac{DS - yRef}{b_0}\right)\right] \tag{2}$$

$$\Delta x \in [LS - xRef, RS - xRef] \tag{3}$$

$$\Delta y \in [DS - yRef, US - yRef] \tag{4}$$

$$\gamma \in \left[-\frac{\pi}{3}, \frac{\pi}{3}\right] \tag{5}$$

$$\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \tag{6}$$

where *RS* represents the reservoir right boundary, *LS* represents the reservoir left boundary, *DS* represents the reservoir down

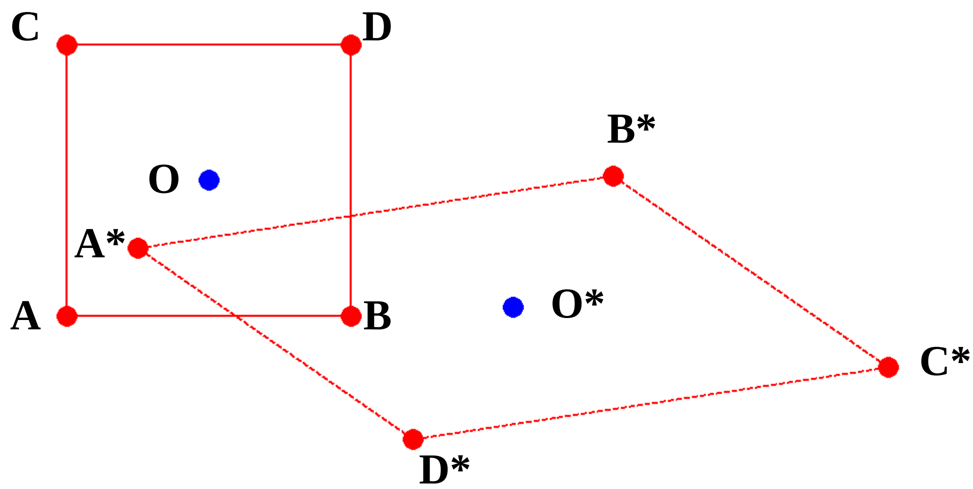


Fig. 5. Transformation sketches of well pattern.

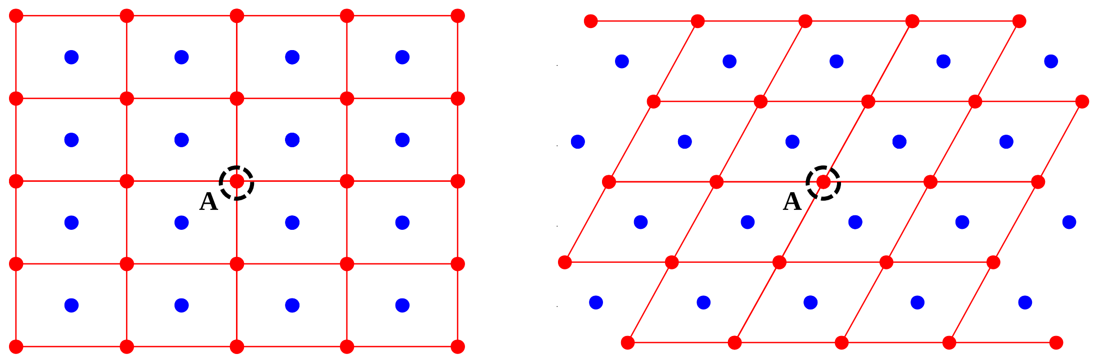


Fig. 6. Sketch of well pattern expansion.

boundary, US represents the reservoir up boundary, $xRef$ represents the x-coordinate of the reference point, $yRef$ represents the y-coordinate of the reference point, a_0 represents the horizontal length of the well pattern, b_0 represents the vertical length of the well pattern.

2.2 Generation of well pattern

Through the above four operators, the new well pattern unit is obtained and then expanded on the whole reservoir to establish the well pattern for the reservoir. Generally, set the reservoir center as the base point, firstly expand the new well pattern in the first quadrant, then the residual quadrants. As shown in Fig. 6, the left graph is the well pattern expansion based on the normal five-spot well pattern unit, while the right one is that based on the new well pattern unit created by the above operators.

3. Well pattern optimization model

3.1 Objective function

The well pattern optimization is to find the maximum or the minimum objective function value over its domain. The optimization problem here is to maximize the NPV by adjusting the well pattern. The NPV is defined by:

$$J(u)=\sum_{n=1}^{N_t}\left\{\frac{\Delta t}{(1+b)^{\frac{t_n}{365}}}\left[\sum_{i=1}^{N_o}\left(r_o Q_{o,i}^n - c_{pw} Q_{pw,i}^n\right)-\sum_{j=1}^{N_w} c_{iw} Q_{iw,j}^n\right]\right\}-\left(N_o C_o + N_w C_w\right) \tag{7}$$

where u is the well pattern vector; N_t denotes the total number of time steps; n denotes the nth time step; Δt_n is the nth time step size, d; the time at the end of nth time step is denoted by t_n; b is the annual discount rate; N_o and N_w denote the number of producers and injectors, repectively; r_o is the oil revenue, RMB/m^3; c_{pw} is the water disposal cost, RMB/m^3; c_{iw} is the water injection cost, RMB/m^3; $Q_{o,i}^n$ and $Q_{pw,i}^n$, respectively, denote the average oil production rate and water production rate at the ith producer for the nth time step, m^3/d; $Q_{iw,j}^n$ is the average injection rate at the jth producer for the nth time

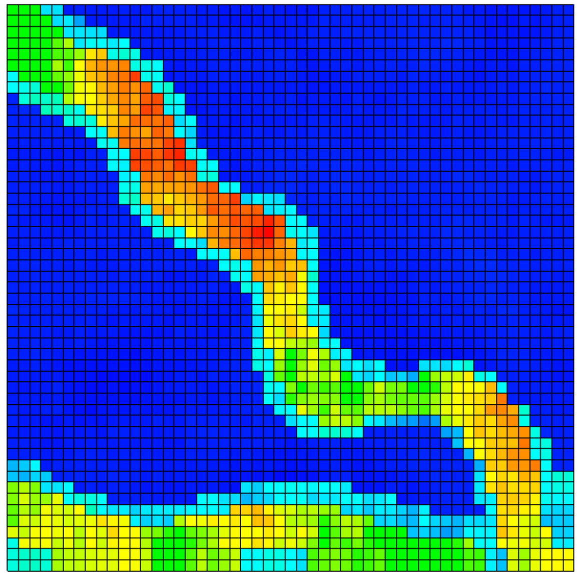


Fig. 7. The reservoir permeability distribution.

Table 1. The cumulative oil production and NPV of different well pattern types.

Type	Cumulative oil production (m^3)		NPV (10^4 RMB)	
	Initial well pattern	Optimized well pattern	Initial well pattern	Optimized well pattern
Five-spot	152,393	185,238	15,474	19,147
Seven-spot	200,250	200,126	14,462	18,188
Nine-spot	152,078	163,187	14,313	17,980

step, m^3/d; C_o and C_w denote the cost of drilling a producer and a injector, respectively, RMB.

3.2 Bound constraints

The value of well pattern vector is limited to a certain extent. The component of well pattern vector should be satisfied with the following condition:

$$u_i^{low} \leq u_i \leq u_i^{up},\ i = 1, 2, \cdots, N_u \tag{8}$$

where u_i denotes the ith component of the well pattern vector; u_i^{low} and u_i^{up} denote the minimum and maximum value of the ith component; N_u denotes the dimension number of the well pattern vector.

The methods dealing with bound constraints include truncation method and the log-transformation method. Here the log-transformation is applied to each element of the well pattern vector. The bound constrained optimization problem can be converted to an unconstrained constrained problem by applying the log-transformation. The ith component of the transformed well pattern vector x corresponding to u is given by:

$$x_i = \ln\left(\frac{u_i - u_i^{low}}{u_i^{up} - u_i}\right) \tag{9}$$

The optimization is done in terms of the transformed well pattern vector, but at each iteration the inversion of the transformed well pattern vector to the original domain is required in order to calculate the objective function value. The well pattern vector u can be obtained by applying the inverse log-transformation to the transformed pattern vector x through the following formula:

$$u_i = \frac{\exp(x_i)\, u_i^{up} + u_i^{low}}{1 + \exp(x_i)} = \frac{u_i^{up} + \exp(-x_i) u_i^{low}}{1 + \exp(-x_i)} \tag{10}$$

Taking the bound constraints into consideration, the well pattern optimization model can be expressed by:

$$Maximize \quad J(u) \tag{11}$$

s.t.

$$u_i^{low} \leq u \leq u_i^{up}, i = 1, 2, \cdots, N_u \tag{12}$$

4. Optimization algorithm

The StoSAG has been successfully applied to the production optimization. The detailed description and derivation of StoSAG was given by Fonseca et al. (2016), so only the calculation procedure is given here. The key to optimization

Table 2. The number of producers and injectors of different well pattern types.

Type	Initial well pattern		Optimized well pattern	
	Producers	Injectors	Producers	Injectors
Five-spot	9	4	6	6
Seven-spot	17	10	14	7
Nine-spot	21	4	9	3

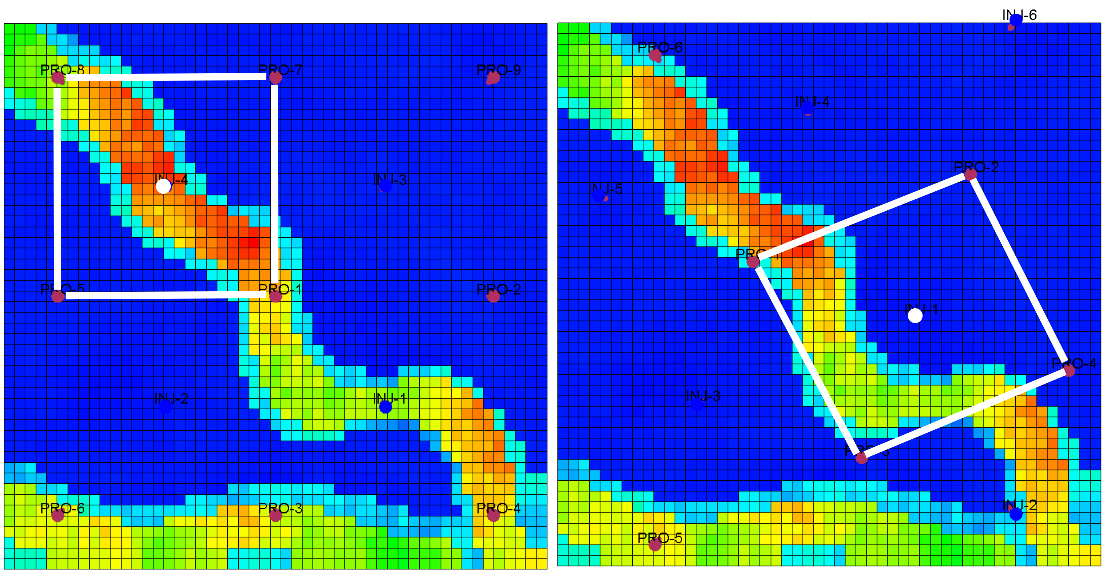
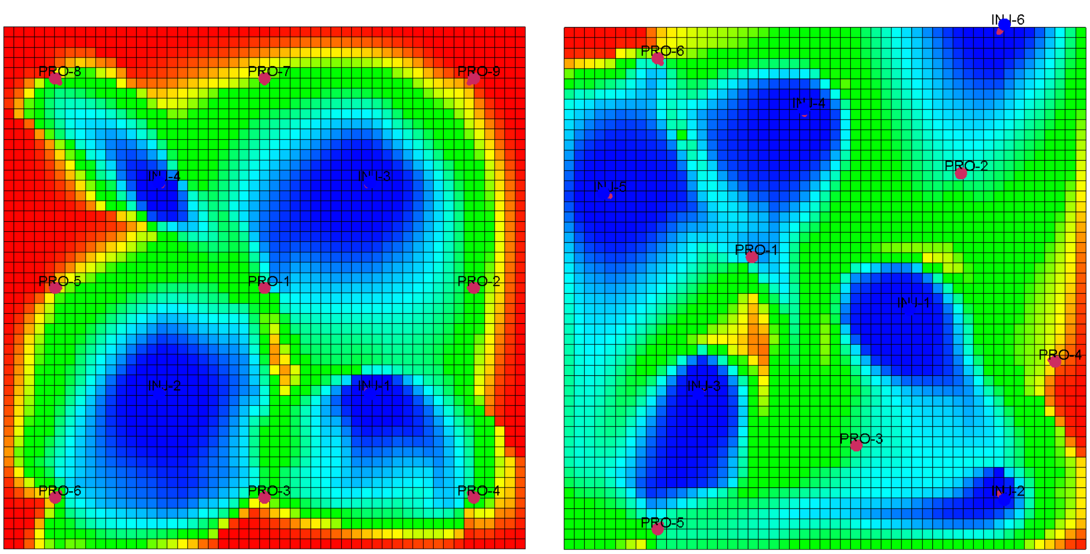


Fig. 8. The initial and optimized well pattern for five-spot (left: initial well pattern; right: optimized well pattern).



(a) (b)

Fig. 9. The oil saturation of initial and optimized well pattern for five-spot (left: initial well pattern; right: optimized well pattern).

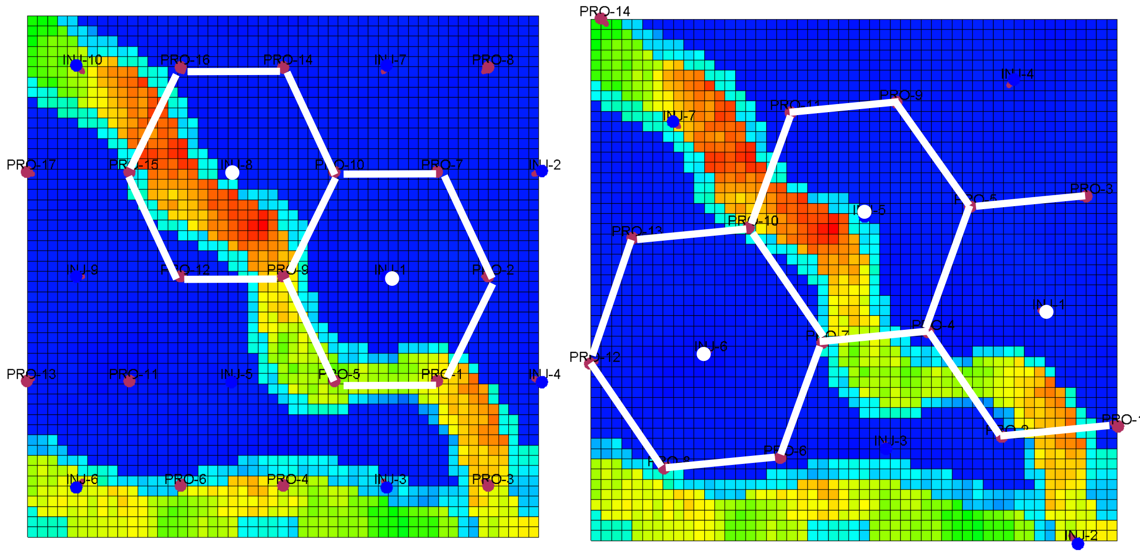
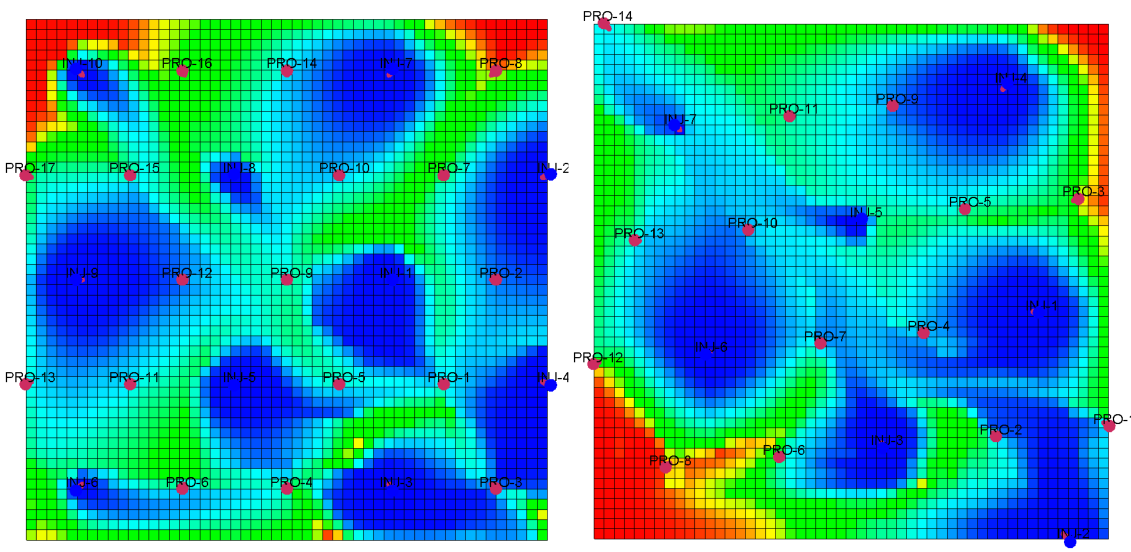


Fig. 10. The initial and optimized well pattern for seven-spot (left: initial well pattern; right: optimized well pattern).



(a) (b)

Fig. 11. The oil saturation of initial and optimized well pattern for seven-spot (left: initial well pattern; right: optimized well pattern).

method is to generate next well pattern vector. For StoSAG, the next well pattern vector is defined as follow:

$$u^{k+1} = u^k + \alpha^k d^k \tag{13}$$

where u^k is the estimate of the optimal well pattern vector at the kth iteration; α^k is the step size obtained by one dimension search method; d^k is the search direction.

The search direction can be determined by the following steps.

Step 1 Generate the covariance matrix C_u by using a spherical covariance function (Oilver et al., 2008). For the element in covariance matrix $C_{i,j}$.

$$C_{i,j} = \sigma^2 \left\{ \begin{array}{c} 1 - \frac{3|i-j|}{2a} + \frac{|i-j|^3}{2a^3},\ |i-j| \leq a \\ 0,\ |i-j| > a \end{array} \right. \tag{14}$$

where i and j denote the control step i and j, respectively; σ refers to the standard deviation; a is the number of correlated control steps.

Step 2 Generate N_e random samples u_i, where $u_i \sim N\left(u^k, C_u\right)$. These samples can be obtained by

$$u_i^k = u^k + C_u^{1/2} Z_i,\ \ i = 1, 2, \cdots, N_e \tag{15}$$

where $C_u^{1/2}$ is the lower triangular matrix obtained by the Cholesky decomposition of C_u; Z_i is the distribution vector, where $Z_i \sim N(0, I)$. I is $N_u \times N_u$ dimension identity matrix.

Step 3 Compute the search direction d^k:

$$d^k = \frac{1}{N_e} \sum_{i=1}^{N_e} \left(u_i^k - u^k\right) \left(J\left(u_i^k\right) - J\left(u^k\right)\right) \tag{16}$$

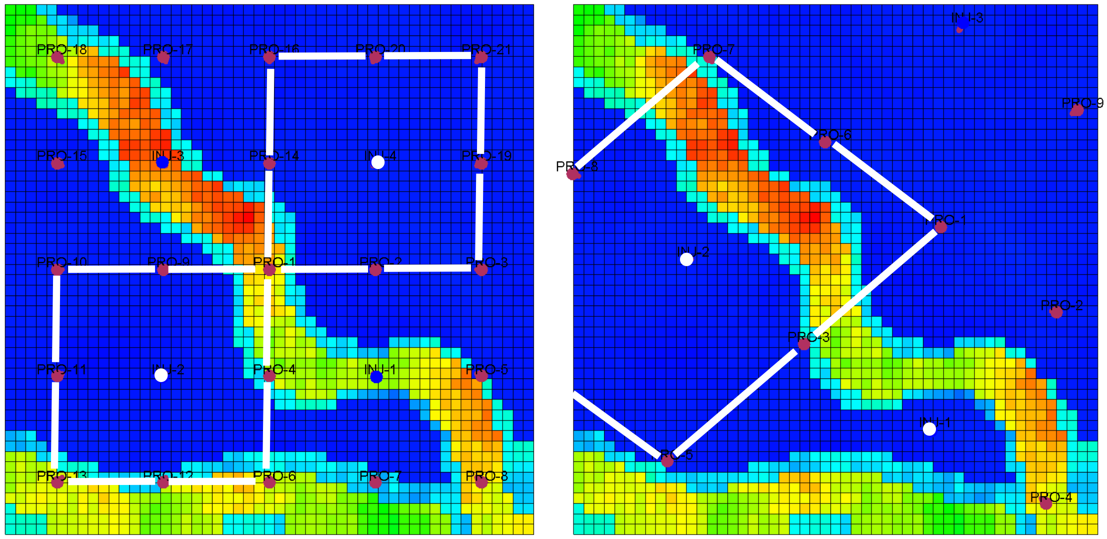
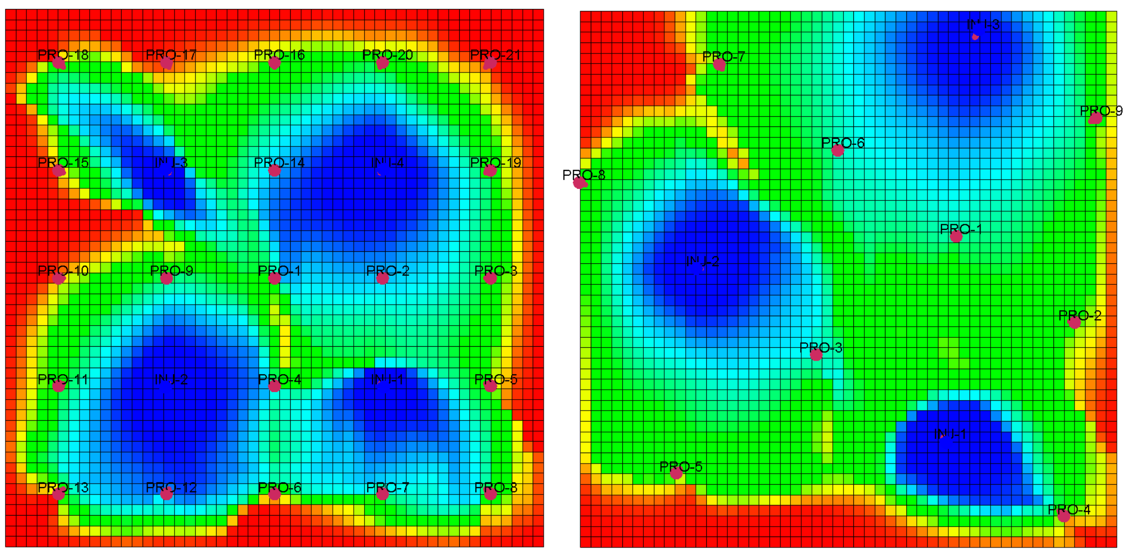


Fig. 12. The initial and optimized well pattern for nine-spot (left: initial well pattern; right: optimized well pattern).



(a) (b)

Fig. 13. The oil saturation of initial and optimized well pattern for nine-spot (left: initial well pattern; right: optimized well pattern).

Step 4 Generate the next well pattern vector u^{k+1}:

$$u^{k+1} = u^k + \alpha^k \frac{d^k}{\|d^k\|_\infty} \tag{17}$$

5. Field application

The method is applied to a simple heterogeneous reservoir to analyze the well placement optimization. The reservoir model contains $51 \times 51 \times 3$ grids. The grid size is 1 0m $\times$ 10 m $\times$ 3 m. The permeability heterogeneity is serious with a high permeability zone in the middle. The permeability distribution is shown is Fig. 7. The porosity is 0.25 and the initial reservoir pressure is 21 MPa. The bottom flowing pressure of the producers is set as 5 MPa. The injectors work at the rate of 80 m^3/d while the maximum injection pressure is not more than 27 MPa. The injection-production rate of the reservoir is 1:1. The total production cycle is 3,600 days. The drilling cost of the producer and injector is 1 million RMB. The oil price is set equal to 2,000 RMB/m^3. The water treatment cost and water injection rate are all 50 RMB/m^3. The annual discount rate is set equal to 12%.

The five-spot, seven-spot and nine-spot well pattern are optimized by the method proposed in this paper. The results are listed in the Tables 1 and 2.

As shown in the Table 1, for the five-spot well pattern, the NPV increases from 154 million RMB to 191 million RMB, an increase of 24%. The cumulative oil production increases from 15.23×10^4 to 18.52×10^4 m^3. The oil saturation of initial and optimized well pattern is shown in the Fig. 9. The development effect is significantly improved. There are 9 producers and 4 injectors in the initial well pattern while there are 6 producers and 6 injectors in the optimized well pattern.

For the seven-spot well pattern, the NPV increases from 144 million RMB to 181 million RMB, an increase of 26%, while the cumulative oil production keeps at the 20×10^4 m^3. The oil saturation of initial and optimized well pattern is shown in the Fig. 11. There are 17 producers and 10 injectors in the initial well pattern while there are 14 producers and 7 injectors in the optimized well pattern. It is obvious that the same cumulative oil production can be obtained by adjusting the location and number of the producers and injector further to improve the economic performance by the optimization.

The NPV of nine-spot well pattern increases from 143 million RMB to 179 million RMB, an increase of 25%. The cumulative oil production increases from 15.21×10^4 to 16.31×10^4 m^3. The oil saturation of initial and optimized well pattern is shown in the Fig. 13. There are 21 producers and 4 injectors in the initial well pattern while there are 9 producers and 3 injectors in the optimized well pattern. By comparison, it can be found that the optimized five-spot well pattern has the best effect. For the development of this oilfield, it is recommended to adopt the optimized five-spot well pattern.

6. Conclusion

The well pattern operators including scaling, shift, shear and rotation are used as the optimization varialbes. Through these well pattern operators, different variations of the well pattern can be obtained. The well pattern optimization is conducted by integrating a numerical simulator with the StoSAG algorithm. The optimal well pattern is obtained by continuously adjusted the well pattern operators by the StoSAG algorithm. The five-spot, seven-spot and nine-spot well pattern are applied in a synthetic reservoir model. The optimization results for the synthetic reservoir model indicates a satisfactory performance of the StoSAG algorithm and the higher NPV value can be obtained by the five-spot well pattern. This method can be applied to the development of new oilfield to provide reference for the construction and adjustment of the well pattern.

Acknowledgments

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