Supplementary file

Effects of pore pressure on coring-induced damage based on simulation by

mesoscale stress-seepage-flow coupling numerical model

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As is known, the mechanical behavior of porous media was first discussed by Biot and Willis, followed by establishing Biot's consolidation theory applicable to soil (Biot and Willis, 1957; Braun et al., 2018). Based on Biot's consolidation theory, many researchers have simulated the hydro-mechanical coupling effect (Prassetyo and Gutierrez, 2016; Li et al., 2020).

Based on the effect of pore pressure, four scenarios can be described, as listed below. These situations have been partly or fully adopted in several numerical simulation software.

(1) No influence

There is no pore pressure that needs to be considered. This also means that the coupling between solid stress and pore pressure can be neglected. Thus, the equilibrium equation of solid deformation can be expressed as:

$$\nabla \cdot \boldsymbol{\sigma} + \mathbf{F}_V = 0 \tag{1}$$

and the virtual work δW can be written as:

$$\delta W = \int_{V}^{\Box} (-\delta \boldsymbol{\varepsilon} \cdot \boldsymbol{\sigma} + \delta \mathbf{u} \cdot \mathbf{F}_{V}) dV + \int_{\Gamma}^{\Box} (\delta \mathbf{u} \cdot \mathbf{F}_{\Gamma}) d\Gamma = 0$$
(2)

where ∇ denotes the Hamiltonian operator; \mathbf{F}_V denotes force per unit volume, N/m³; ε denotes total strains; *V* denotes volume, m³; Γ denotes boundary, m²; \mathbf{F}_{Γ} denotes the force acting on the boundary, N/m²; δW denotes virtual work, J; $\delta \varepsilon$ denotes virtual strains and $\delta \mathbf{u}$ denotes virtual displacements, m; σ denotes the stresses of solid structure, Pa, which can be determined by.

$$\boldsymbol{\sigma} = \mathbf{C} : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_{ie}) \tag{3}$$

where C represents the fourth order elasticity tensor, Pa; ϵ_{ie} refers to the inelastic strains.

(2) Additive stress

Pore pressure is considered as an additive stress contribution that has a source other than the constitutive relation of a solid structure. In this situation, the equilibrium equation and the virtual work is still written as in Eqs. (1) and (2) respectively, but the stresses of solid structure σ need to be described as:

$$\boldsymbol{\sigma} = \mathbf{C}: \left(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_{ie}\right) + \alpha_B P \mathbf{I} \tag{4}$$

where α_B denotes the Biot's coefficient, whose value can be set as 0.3 for most rocks (Zhu et al., 2011); *P* denotes pore pressure, Pa; **I** is the identity tensor. This means that the compressive stresses are augmented due to the existence of pore pressure. The pore pressure and compression stress are both expressed as positive values in this paper.

(3) Effective stress

The pore pressure is treated as a volume load acting on the solid structure to balance external loads, whereas the pore pressure is not added into the stress tensor. Thus, the equilibrium equation can be expressed as in Eq. (5) and the virtual work is written as:

 $\nabla \cdot (\mathbf{\sigma} + \alpha_B P \mathbf{I}) + \mathbf{F}_V = 0 \tag{5}$

$$\delta W = \int_{V}^{\Box} (-\delta \boldsymbol{\varepsilon} \cdot \boldsymbol{\sigma} + \delta \mathbf{u} \cdot \mathbf{F}_{V} - \delta \boldsymbol{\varepsilon} \cdot \boldsymbol{\alpha}_{B} P \mathbf{I}) dV + \int_{\Gamma}^{\Box} (\delta \mathbf{u} \cdot \mathbf{F}_{\Gamma}) d\Gamma = 0$$
(6)

where the stress σ is often called as effective stress. It is carried by the solid structure excluding the pore pressure and can be determined by the constitutive equation as indicated in Eq. (3). In this situation, there is no contribution to the stress tensor, and the only effect of the pore pressure is acting as a load. It can be noticed that the virtual work from Eq. (6) is the same as that from Eq. (2) after substituting Eq. (4) into Eq. (2), that is:

$$\delta W = \int_{V}^{\Box} (-\delta \boldsymbol{\epsilon}: (\boldsymbol{\epsilon} - \boldsymbol{\epsilon}_{ie}) + \alpha_{B} P \mathbf{I}) + \delta \mathbf{u} \cdot \mathbf{F}_{V}) dV + \int_{\Gamma}^{\Box} (\delta \mathbf{u} \cdot \mathbf{F}_{\Gamma}) d\Gamma = 0$$
(7)

Therefore, when the pore pressure is considered as either a volume load or an additive stress, the elastic solid deformation will be the same. The only difference for these two situations is the stress value, which will impact on the inelastic strains such as plastic, damage, creep, and so on.

(4) Residual stress

In this situation, there are contributions both to the stress tensor and body load. The stress tensor is prescribed as in Eq. (4) to reflect the effect of pore pressure, and the equilibrium equation is expressed as in Eq. (1). In the meantime, an external load with opposite sign is applied to remove deformation from pore pressure. Accordingly, the virtual work is written as:

$$\delta W = \int_{V}^{\square} (-\delta \boldsymbol{\varepsilon} \cdot \boldsymbol{\sigma} + \delta \mathbf{u} \cdot \mathbf{F}_{V} + \delta \boldsymbol{\varepsilon} \cdot \boldsymbol{\alpha}_{B} P \mathbf{I}) dV + \int_{\Gamma}^{\square} (\delta \mathbf{u} \cdot \mathbf{F}_{\Gamma}) d\Gamma = 0$$
(8)

Therefore, Eq. (9) can be derived by substituting Eq. (4) into Eq. (8).

$$\delta W = \int_{V}^{\Box} (-\delta \boldsymbol{\varepsilon} : \mathbf{C} : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_{ie}) + \delta \mathbf{u} \cdot \mathbf{F}_{V}) dV + \int_{\Gamma}^{\Box} (\delta \mathbf{u} \cdot \mathbf{F}_{\Gamma}) d\Gamma = 0$$
(9)

This means that the deformation due to pore pressure has been eliminated. Thus, pore pressure is considered as a residual pressure that augments the stress tensor but does not create any initial strains.



Fig. S1. Boundary conditions of numerical models.

Table S1 lists the stress formula, the equilibrium equation and the virtual equation under these four kinds of scenarios. To illustrate their differences, a simple numerical model (Fig. S1) was calculated. A 1/8 sphere with a radius of 500 μm was established to simulate a porous medium regarded as a coupling of solid-deformation and fluid-seepage. Its elasticity modulus was set as 35.0 GPa and its Poisson's ratio as 0.25. All three inner surfaces were set as a symmetric boundary condition. The outer sphere surface was set as a varying boundary condition: (1) Free boundary, i.e., $P_r = 0$; (2) fixed boundary, i.e., $u_r = 0$; (3) constant pressure, i.e., $P_r = 20$ MPa; (4) constant displacement, i.e., a radial displacement was given as 0.143 µm corresponding to the external pressure of 20.0 MPa. The pore pressure was set to be 10.0 MPa and the Biot's coefficient was 0.3. The pore pressure effect was split into four different situations as discussed above, and the results under various boundary conditions were listed in Table S1.

| Class | Stress formula | Equilibrium equation | Virtual equation | Numerical simulation and results | | | | |
|---------------------|-------------------|-------------------------|---------------------|----------------------------------|-----------------------|----------------------------|--------------------------------|-------------------------------|
| | | | | Boundary conditions ¹ | Total stress (MPa) | Elastic stress (MPa) | Radial displacement (µm) | Radial Elastic strain (με) |
| No influence | Eq.(3) | Eq.(1) | Eq.(2) | Free $(P_r = 0)$ | 0 | 0 | 0 | 0 |
| | | | | Fixed $(u_r = 0)$ | 0 | 0 | 0 | 0 |
| | | | | $P_{\rm r} = 20 {\rm ~MPa}$ | 20 | 20 | 0.143 | 286 |
| | | | | $u_{\rm r} = 0.143 \ \mu{\rm m}$ | 20 | 20 | 0.143 | 286 |
| Additive stress | Eq.(4) | Eq.(1) | Eq.(2) | Free $(P_r = 0)$ | 0 | -3 | -0.021 | -43 |
| | | | | Fixed $(u_r = 0)$ | 3 | 0 | 0 | 0 |
| | | | | $P_{\rm r} = 20 {\rm ~MPa}$ | 20 | 17 | 0.121 | 243 |
| | | | | $u_{\rm r} = 0.143 \ \mu{\rm m}$ | 23 | 20 | 0.143 | 286 |
| Effective stress | Eq.(3) | Eq.(5) | Eq.(6) | Free $(P_r = 0)$ | -3 | -3 | -0.021 | -43 |
| | | | | Fixed $(u_r = 0)$ | 0 | 0 | 0 | 0 |
| | | | | $P_{\rm r} = 20 {\rm MPa}$ | 17 | 17 | 0.121 | 243 |
| | | | | $u_{\rm r} = 0.143 \ \mu{\rm m}$ | 20 | 20 | 0.143 | 286 |
| Residual stress | Eq.(4) | Eq.(1) | Eq.(9) | Free $(P_r = 0)$ | 3 | 0 | 0 | 0 |
| | | | | Fixed $(u_r = 0)$ | 3 | 0 | 0 | 0 |
| | | | | $P_{\rm r} = 20 {\rm ~MPa}$ | 23 | 20 | 0.143 | 286 |
| | | | | $u_{\rm r} = 0.143 \ \mu{\rm m}$ | 23 | 20 | 0.143 | 286 |

Table S1 Effect of pore pressure under different considerations.

Notes: $^{1}P_{r}$ refers to the pressure applied to the outer sphere surface; u_{r} is the displacement given on the outer sphere surface.

The results show that when no pore pressure is considered, the results from constant pressure or constant displacement boundary are equivalent. If pore pressure is regarded as either an additive stress or an effective stress, the results from constant pressure and constant displacement boundary are distinct because of the effect of pore pressure on deformation. According to Eq. (7), the elastic deformation in these two situations is equivalent, which can also be validated from the numerical results. Under prescribed constant displacements, determining the deformation of solid is unaffected, and the pore pressure impacts only on the stress. Therefore, the elastic stress and strain remain same to that without any pore pressure present. The effective stress is equal to the elastic stress, while the additive stress is the sum of elastic stress and pore pressure. Under constant outer pressure loading, the pore pressure can supply to balance part of the external loading, thus decreasing the final elastic deformation of the solid. Therefore, the elastic stress and strain become smaller than those without any pore pressure, and the effective stress and the additive stress accordingly become smaller. On the other hand, if pore pressure is regarded as a residual stress, the total stress becomes larger due to the addition of pore pressure, while the deformation of solid remains consistent under either constant pressure or constant displacement boundary. Such situation can well reflect geostress in rock engineering, in which rock masses are subject to vertical overburden stress and horizontal confining stress, and their initial deformation due to geostress is usually dismissed.

References

- Biot, M. A., Willis, D. G. The elastic coefficients of the theory of consolidation. Journal of Applied Mechanics, 1957, 24(4): 594-601.
- Braun, P., Ghabezloo, S., Delage, P., et al. Theoretical analysis of pore pressure diffusion in some basic rock mechanics experiments. Rock Mechanics and Rock Engineering, 2018, 51(5): 1361-1378.
- Li, M., Guo, P., Stolle, D. F. E., et al. Modeling hydraulic fracture in heterogeneous rock materials using permeabilitybased hydraulic fracture model. Underground Space, 2020, 5(2): 167-183.
- Prassetyo, S. H., Gutierrez, M. Effect of surface loading on the hydro-mechanical response of a tunnel in saturated ground. Underground Space, 2016, 1(1): 1-19.
- Zhu, H., Liu, Q., Wong, G., et al. A pressure and temperature preservation system for gas-hydrate-bearing sediments sampler. Petroleum Science and Technology, 2013, 31(6): 652-662.